Handout 1.7

- A ***theorem*** is a statement that can be shown to be true using:
  - definitions
  - other theorems
  - ***axioms*** (statements which are given as true)
  - rules of inference
- A ***lemma*** is a 'helping theorem' or a result which is needed to prove a theorem.
- A ***corollary*** is a result which follows directly from a theorem.
- Less important theorems are sometimes called ***propositions***.
- A ***conjecture*** is a statement that is being proposed to be true. Once a proof of a conjecture is found, it becomes a theorem. It may turn out to be false.
- Many theorems have the form:

$$\forall x(P(x) \to Q(x))$$

- To prove them, we show that where c is an arbitrary element of the domain,

$$P(c) \to Q(c)$$

- By universal generalization the truth of the original formula follows
- So, we must prove something of the form: $p \to q$

- ***Direct Proof*:** Assume that p is true. Use rules of inference, axioms, and logical equivalences to show that q must also be true.
- ***Proof by Contraposition*:** Assume $\neg q$ and show $\neg p$ is true also. This is sometimes called an ***indirect proof*** method. If we give a direct proof of $\neg q \to \neg p$ then we have a proof of $p \to q$.
- ***Proof by Contradiction*:** (AKA *reductio ad absurdum*).

  To prove p, assume $\neg p$ and derive a contradiction such as $p \wedge \neg p$. (an indirect form of proof). Since we have shown that $\neg p \to F$ is true , it follows that the contrapositive $\mathbf{T} \to p$ also holds.