Handout 1.6

- An ***argument*** in propositional logic is a sequence of propositions. All but the final proposition are called ***premises***. The last statement is the ***conclusion***.
- The argument is valid if the premises imply the conclusion. An ***argument form*** is an argument that is valid no matter what propositions are substituted into its propositional variables.
- If the premises are $p_1, p_2, \ldots, p_n$ and the conclusion is q then $(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \rightarrow q$ is a tautology.
- Inference rules are all argument simple argument forms that will be used to construct more complex argument forms.

Modus Ponens:

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

Corresponding Tautology: $(p \wedge (p \rightarrow q)) \rightarrow q$

Modus Tollens:

$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Corresponding Tautology: $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$

Hypothetical Syllogism:

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

Corresponding Tautology: $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

Disjunctive Syllogism:

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

Corresponding Tautology: $(\neg p \wedge (p \vee q)) \rightarrow q$

Addition:

$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$$

Corresponding Tautology: $p \rightarrow (p \vee q)$

Simplification:

$$\frac{p \wedge q}{\therefore q}$$

Corresponding Tautology: $(p \wedge q) \rightarrow p$

Conjunction:

$$\frac{\begin{array}{l} p \\ q \end{array}}{\therefore p \wedge q}$$

Corresponding Tautology: $((p) \wedge (q)) \rightarrow (p \wedge q)$

Resolution:

$$\frac{\begin{array}{l} \neg p \vee r \\ p \vee q \end{array}}{\therefore q \vee r}$$

Corresponding Tautology: $((\neg p \vee r) \wedge (p \vee q)) \rightarrow (q \vee r)$

Universal Instantiation (UI):

$$\frac{\forall x P(x)}{\therefore P(c)}$$

Universal Generalization (UG):

$$\frac{P(c) \text{ for an arbitrary } c}{\therefore \forall x P(x)}$$

Existential Instantiation (EI):

$$\frac{\exists x P(x)}{\therefore P(c) \text{ for some element } c}$$

Existential Generalization (EG):

$$\frac{P(c) \text{ for some element } c}{\therefore \exists x P(x)}$$