1.3 Prop. Equivalences, 1.4 Predicates and Quantifiers and 1.5 Nested Quantifiers - Worksheet

1. Five friends have access to a chat room. Is it possible to determine who is chatting if the following information is known: Either Kevin or Heather, or both, are chatting. Either Randy or Vijay, but not both, are chatting. If Abby is chatting, so is Randy. Vijay and Kevin are either both chatting or neither is. If Heather is chatting, then so are Abby and Kevin. Explain your reasoning.

2. Show that $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$ are logically equivalent.

3. Use logical equivalences to show that $(p \wedge q) \to (p \vee q)$ is a tautology.

4. Let $C(x)$ be the statement "x has a cat'," let $D(x)$ be the statement "x has a dog," and let $F(x)$ be the statement "x has a ferret." Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiiers and logical connectives. Let the domain consist of all students in your class.

   (a) A student in your class has a cat, a dog and a ferret.

   (b) All students in your class have a cat, a dog, or a ferret.

   (c) Some student in your class has a cat and a ferret, but not a dog.

   (d) No student in your class has a cat, a dog and a ferret.

   (e) For each of the three animals, cats, dogs and ferrets, there is a student in your class who has this animal as a pet.

5. Determine the truth value of each of these statements if the doman consists of all integers.

   (a) $\exists x(x^3 = -1)$

   (b) $\exists x(x^4 < x^2)$

   (c) $\forall x((-x)^2 = x^2)$

   (d) $\forall x(2x > x)$

6. Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all real numbers.

   (a) $\forall x(x^2 \neq x)$

   (b) $\forall x(x^2 \neq 2)$

   (c) $\forall x(|x| > 0)$

7. Let $F(x, y)$ be the statment "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statments.

   (a) Everybody can fool Fred.

(b) Evelyn can fool everybody.
(c) Everybody can fool somebody.
(d) There is no one who can fool everybody.
(e) Everyone can be fooled by somebody.
(f) No one can fool both Fred and Jerry.

8. Express the negations of each of these statements so that all negation symbols immediately precede predicates.

   (a) $\exists z \forall y \forall x T(x, y.z)$
   (b) $\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$
   (c) $\exists x \exists y (Q(x, y) \leftrightarrow Q(y, x))$
   (d) $\forall y \exists x \exists z (T(x, y, z) \vee Q(x, y))$