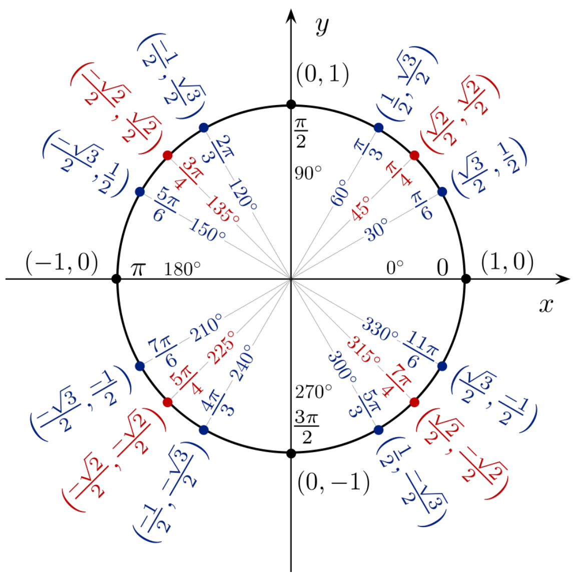
Mathematics 1375/D572, Spring 2019 **Exam #4 (Take-home)**
Instructor: Suman Ganguli **Due: Friday, May 17** **Name:** ____________________

Question:	1	2	3	4	5	6	7	8	Total
Points:	3	12	5	10	5	5	5	5	50
Score:									

In order to receive full credit, you must **show all your work** and simplify your answers. You must **work on this exam alone**, but you should study resources such as your class notes, WebWork exercises, the Final Exam Review exercises, the textbook, etc.

1. (3 points) Recall the following graph of the unit circle, which shows the coordinates of points on the unit circle at certain "special angles":



(a) We can use this graph of the unit circle to find solutions of *some* trigonometric equations of the form

$\sin\theta = c$ or $\cos\theta = c$

For which values of c will this graph work?

$c =$

(b) Describe, step by step, the procedure by which we can use this graph of the unit circle to find *all* solutions of such trigonometric equations $\sin\theta = c$ or $\cos\theta = c$:

2. (12 points) Find all solutions (in radians) of the following trigonometric equations. For each equation:

- show all preliminary algebraic steps
- explain how you use the unit circle to find the solutions in $[0, 2\pi)$
- and then write down expressions giving all solutions

(a) $8\sin\theta + 4 = 0$

(b) $\tan^2\theta - 1 = 0$ (Hint: use $u^2 - 1 = (u-1)(u+1)$.)

(c) $2\cos^2\theta - \cos\theta = 0$ (Hint: use $2u^2 - u = u(2u-1)$.)

3. (5 points) The mathematical model for exponential growth or decay of some quantity Q is given by the function

$$Q(t) = Q_0(1+r)^t$$

where the independent variable t typically represents time.

(a) What do the parameters (i.e., constants) in the function $Q(t)$ represent?

- Q_0:
- r:

(b) What determines whether Q is undergoing exponential growth or exponential decay? (Hint: your explanation should use simple inequalities involving one of the parameters.)

4. (10 points) The world's population is approximately 7 billion people at present, and is estimated to be growing at a rate of 1.2% per year.

(a) Assuming the growth rate stays constant, write down the formula for $P(t)$ = the world population t years from now:

$P(t) =$

(b) Using these assumptions, what will the world population be in 2050 (i.e., 31 years from now)?

(c) Under these assumptions, in what year will the world population be double what it is now? Show all algebra.

5. (5 points) Consider a trigonometric function of the form $f(x) = a\sin(bx + c)$ or $f(x) = a\cos(bx + c)$.

   Give the formula for calculating each of the following quantities (in terms of the parameters a, b, c) and briefly describe what each quantity represents on the graph of the trig function:

   - amplitude of $f =$

     The ampltitude represents:

   - period of $f =$

     The period represents:

   - phase shift:

     The phase shift represents:

6. (5 points) Consider the function $f(x) = -5\sin(2x - \pi)$.

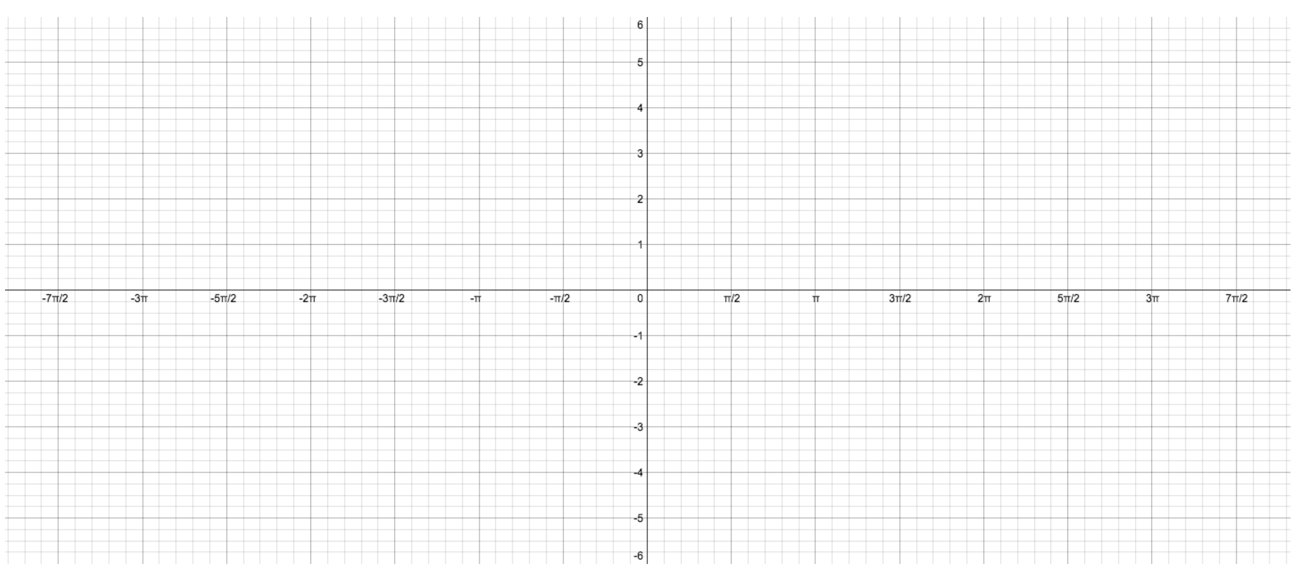
   (a) Find the amplitude, period and phase shift of $f(x)$:

   - amplitude:

   - period:

   - phase shift:

   (b) Sketch the graph of $f(x)$. Label the amplitude, period and phase shift on the graph.



7. (5 points) An arithmetic sequence is a sequence of numbers $a_1, a_2, a_3, a_4, \ldots$ where there is a constant "difference" d between successive terms, i.e.,

$$a_2 = a_1 + d, a_3 = a_2 + d, \ldots$$

The general formula for the n-th term in an arithmetic sequence is:

$$a_n = a_1 + d(n-1)$$

and the formula for the sum of the first k terms in an arithmetic sequence is:

$$a_1 + a_2 + a_3 + \ldots + a_k = \frac{k(a_1 + a_k)}{2}$$

Now consider the arithmetic sequence

$$a_1 = 4, a_2 = 7, a_3 = 10, a_4 = 13, \ldots$$

(a) Apply the general formula to calculate the 40th term in this arithmetic sequence (i.e., you should **not** calculate all the terms $a_5, a_6 \ldots a_{39}$!)

$$a_{40} =$$

(b) Apply the formula to find the sum of the first 40 terms in this arithmetic sequence:

$$a_1 + a_2 + a_3 + \ldots + a_{40} =$$

8. (5 points) A geometric sequence is a sequence of numbers $a_1, a_2, a_3, a_4, \ldots$ where there is a constant "ratio" r between successive terms, i.e.,

$$a_2 = a_1 * r, a_3 = a_2 * r, \ldots$$

For a geometric series with $-1 < r < 1$, the infinite geometric sum is given by the formula

$$a_1 + a_2 + a_3 + \ldots = \frac{a_1}{1-r}$$

Now consider the geometric sequence

$$a_1 = 5, a_2 = -\frac{5}{3}, a_3 = \frac{5}{9}, a_4 = -\frac{5}{27}, \ldots$$

(a) For this geometric sequence, the ratio r is

$$r =$$

(b) Calculate the sum of this infinite geometric sequence:

$$a_1 + a_2 + a_3 + \ldots =$$