Mathematics 1375/D572, Spring 2019 **Exam #1**
Instructor: Suman Ganguli **Friday, February 22** **Name:** ____________________

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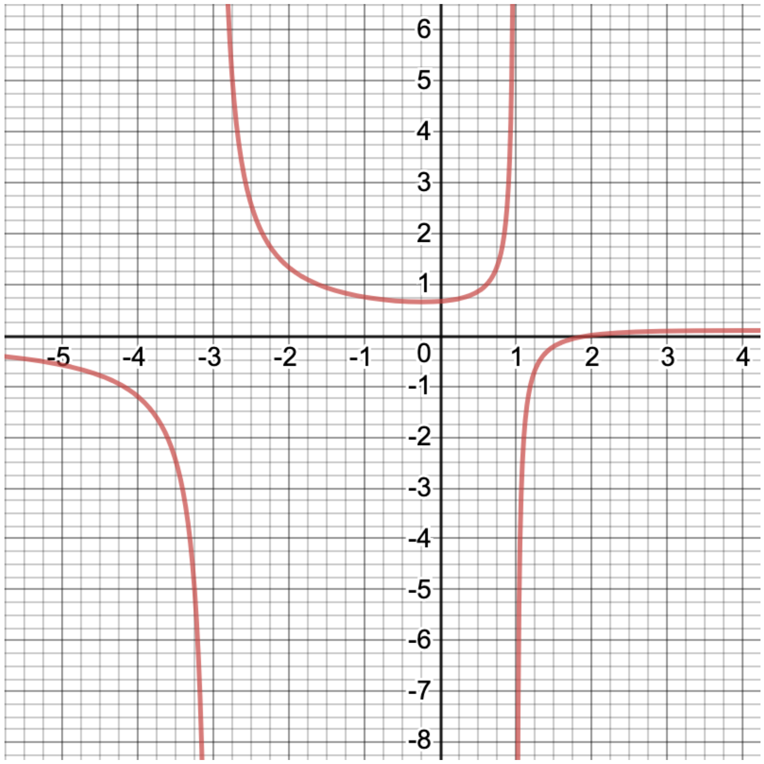
1. (10 points) Shown is the graph of the function $f(x) = \dfrac{x-2}{x^2+2x-3}$:



(a) Compute the following values of f (show your calculations), and **label the corresponding points with their coordinates** on the graph above:

Solution:

- $f(0) = \dfrac{0-2}{0+0-3} = \dfrac{2}{3}$
- $f(2) = \dfrac{2-2}{4+4-3} = 0$
- $f(-4) = \dfrac{-4-2}{16-8-3} = -\dfrac{6}{5}$

(b) What is the domain of f? For full credit, write the solution in interval notation. (Hint: Start by factoring the denominator.)

Solution: Since the denominator of f is $x^2+2x-3 = (x+3)(x-1)$, the function is undefined for $x = -3$ and $x = 1$. Hence, the domain of f is

$$(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$$

2. (10 points) Solve each of the following inequalities algebraically, and

- write the solution set in interval notation
- graph the solution set on the given number line

(a) $|3 - 2x| > 7$

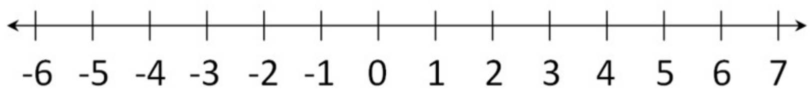
Solution:

$3 - 2x < -7 \quad \text{or} \quad 3 - 2x > 7$

$-2 < -10 \quad \text{or} \quad -2x > 4$

$x > 5 \quad \text{or} \quad x < -2$

$(-\infty, -2) \cup (5, \infty)$



(b) $|4x - 3| \leq 5$

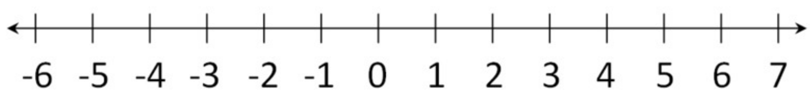
Solution:

$-5 \leq 4x - 3 \leq 5$

$-2 \leq 4x \leq 8$

$-\frac{1}{2} \leq x \leq 2$

$\left[-\frac{1}{2}, 2\right]$



3. (10 points) Write down **and simplify** the following for $g(x) = x^2 - 7x - 20$:

(a) $g(x+h) =$

Solution: $g(x+h) = (x+h)^2 - 7(x+h) - 20 = x^2 + 2xh + h^2 - 7x - 7h - 20$

(b) $g(x+h) - g(x) =$

Solution: $g(x+h) - g(x) = (x^2 + 2xh + h^2 - 7x - 7h - 20) - (x^2 - 7x - 20) = 2xh + h^2 - 7h$

(c) $\dfrac{g(x+h) - g(x)}{h} =$

Solution: $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2xh + h^2 - 7h}{h} = 2x + h - 7$

4. (10 points) Let $f(x) = 4x - 1$ and $g(x) = \sqrt{x}$. Write down and simplify expressions for the following functions, and find their respective domains.

(a) $\left(\frac{f}{g}\right)(x) =$

domain of $\left(\frac{f}{g}\right)$:

Solution: $\left(\frac{f}{g}\right)(x) = \dfrac{4x-1}{\sqrt{x}}$ domain: $(0, \infty)$

(b) $\left(\frac{g}{f}\right)(x) =$

domain of $\left(\frac{g}{f}\right)$:

Solution: $\left(\frac{g}{f}\right)(x) = \dfrac{\sqrt{x}}{4x-1}$ domain: $[0, 1/4) \cup (1/4, \infty)$

(c) $(f \circ g)(x) =$

domain of $(f \circ g)$:

Solution: $(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = 4\sqrt{x} - 1$ domain: $[0, \infty)$

(d) $(g \circ f)(x) =$

domain of $(g \circ f)$:

Solution: $(g \circ f)(x) = g(f(x)) = g(4x - 1) = \sqrt{4x-1}$ domain: $[1/4, \infty)$

5. (10 points) Find the inverse $f^{-1}(x)$ of the function $f(x) = \dfrac{4}{x-3}$

(Recall: to find $f^{-1}(x)$, start by setting up the equation $y = f(x)$ and then solve for x in terms of y.)

Solution: We set up the equation $y = f(x)$:

$$y = \frac{4}{x-3}$$

and solve for x in terms of y:

$$y(x-3) = 4 \Longrightarrow x - 3 = \frac{4}{y} \Longrightarrow x = \frac{4}{y} + 3$$

Swapping x and y, we get $y = \dfrac{4}{x} + 3$, and so:

$$f^{-1}(x) = \frac{4}{x} + 3$$

Check: $f(f^{-1}(x)) = \dfrac{4}{\frac{4}{x} + 3 - 3} = \dfrac{4}{\frac{4}{x}} = x$