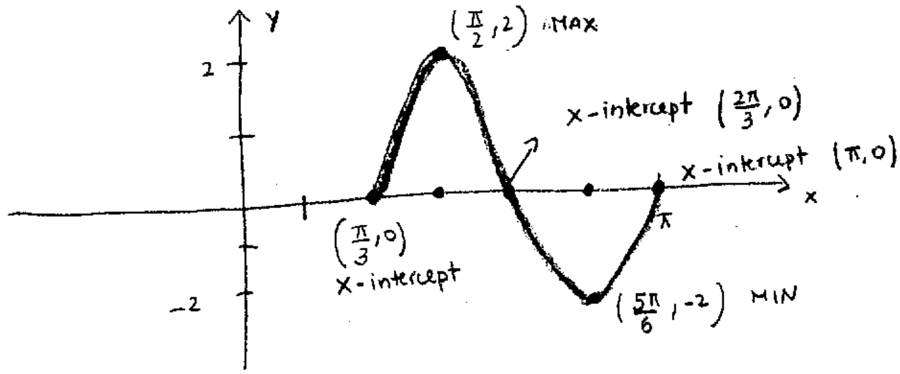
① $y = 2\sin(3x - \pi)$ $\quad A = 2 \quad B = 3 \quad C = -\pi$

Amplitude = 2 $\qquad$ phase shift $= -\frac{C}{B} = \frac{\pi}{3}$

Period $= \left|\frac{2\pi}{B}\right| = \frac{2\pi}{3}$

cycle starts at $\frac{\pi}{3}$ (phase shift) and ends at $\frac{\pi}{3} + \frac{2\pi}{3} = \pi$ (phase shift + period)

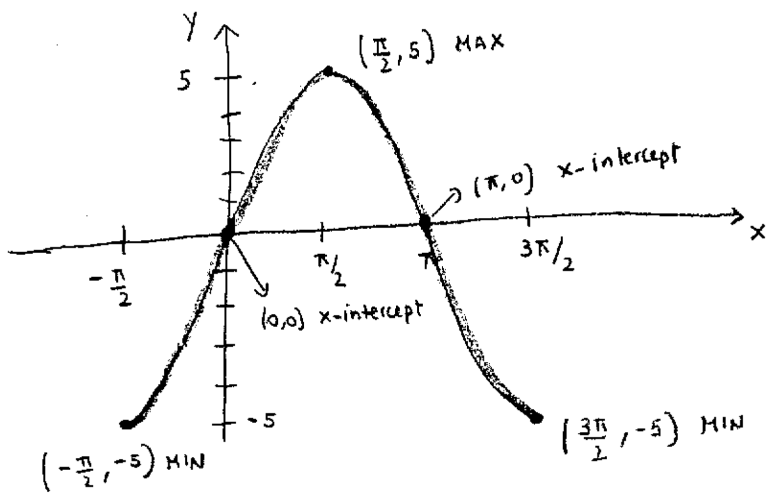


② $y = -5\cos\left(x + \frac{\pi}{2}\right)$ $\quad A = -5 \quad B = 1 \quad C = \frac{\pi}{2}$

Amplitude $= |A| = 5$ $\qquad$ phase shift $= -\frac{C}{B} = -\frac{\pi}{2}$

Period $= \frac{2\pi}{1} = 2\pi$

cycle starts at $-\frac{\pi}{2}$ and ends at $-\frac{\pi}{2} + 2\pi = \frac{3\pi}{2}$



③ $-2\log x + \frac{1}{2}\log y - 4\log z = -\log x^2 + \log y^{\frac{1}{2}} - \log z^4 =$

$-\log x^2 + \log\sqrt{y} - \log z^4 = \boxed{\log\left(\frac{\sqrt{y}}{x^2 z^4}\right)}$

(4) $P(t) = 35.7\,(1+0.01)^t$ where $t=0$ is the year 2010

population in 2015 $= P(5) = 35.7\,(1.01)^5 = 37.52105879 = \boxed{37.5 \text{ million}}$

$35.7 * 2 = 71.4$. We need to find in what year the population will be 71.4 million

$71.4 = 35.7\,(1.01)^t$ divide by 35.7

$2 = (1.01)^t$

$\ln 2 = \ln (1.01)^t$

$\ln 2 = t \ln(1.01)$ $\quad t = \dfrac{\ln 2}{\ln(1.01)} = 69.66071689$

So the population will be double sometimes in the $\boxed{\text{year } 2079}$

(5) $Q(t) = 45\,e^{rt}$ We need to find r using that $Q(3) = 22.5$ (half of 45)

$\dfrac{22.5}{45} = \dfrac{45}{45} e^{r(3)}$ $\quad \dfrac{1}{2} = e^{3r}$

$\ln\left(\frac{1}{2}\right) = \ln\left(e^{3r}\right) = 3r \ln(e) = 3r$ $\quad r = \dfrac{\ln\left(\frac{1}{2}\right)}{3} = -.2310490602$

$\boxed{Q(t) = 45\,e^{-.231049t}}$

$7 = 45\,e^{-.231049t}$ $\quad \dfrac{7}{45} = e^{-.231049t}$

$\ln\left(\frac{7}{45}\right) = -.231049t$ $\quad t = \dfrac{\ln\left(\frac{7}{45}\right)}{-.231049} = 8.053496621$

$\boxed{8.05 \text{ hours}}$

(6) $\log_3(x) + \log_3(x-8) = 2$

$\log_3(x(x-8)) = 2$

$\log_3(x^2 - 8x) = 2$

$x^2 - 8x = 3^2 = 9$

$x^2 - 8x = 9$

$x^2 - 8x - 9 = 0$

$(x-9)(x+1) = 0$

$\boxed{x=9}$

$\cancel{x=-1}$ reject since inputs of log are positive

(7) $f(x) = -\log(3-2x)$

Domain $3 - 2x > 0$

$-2x > -3$

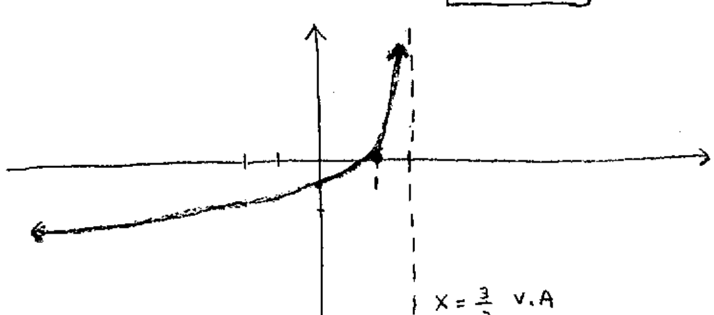
$x < \frac{3}{2}$ $\quad \boxed{\left(-\infty, \frac{3}{2}\right)}$

V.A. $\boxed{x = \frac{3}{2}}$

x-intercept $3 - 2x = 1$

$-2x = -2$

$x = 1$ $\quad \boxed{(1,0)}$



(8) $\ln\sqrt{\dfrac{xy^3}{\sqrt{z}}} = \ln\left(\dfrac{xy^3}{z^{1/2}}\right)^{1/2} = \ln\left(\dfrac{x^{1/2}\,y^{3/2}}{z^{1/4}}\right)$

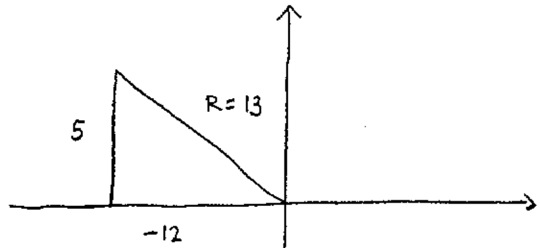
$= \ln x^{1/2} + \ln y^{3/2} - \ln z^{1/4} = \frac{1}{2}\ln x + \frac{3}{2}\ln y - \frac{1}{4}\ln z = \boxed{\frac{1}{2}u + \frac{3}{2}v - \frac{1}{4}w}$

(9) $\tan(\alpha) = -\frac{5}{12} = \frac{O}{A}$

$5^2 + (-12)^2 = R^2$

$25 + 144 = 169 \quad R = 13$

$\sin(\alpha) = \frac{5}{13} \qquad \cos(\alpha) = -\frac{12}{13}$



$\sin(2\alpha) = 2\sin\alpha\cos\alpha = 2\cdot\frac{5}{13}\cdot\left(-\frac{12}{13}\right) = \boxed{\frac{-120}{169}}$

$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = \left(-\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2 = \frac{144-25}{169} = \boxed{\frac{119}{169}}$

(10) a) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$ $\qquad \boxed{x = \frac{\pi}{6} + N\pi \quad N = 0, \pm1, \pm2, \dots}$

b) $\cos^{-1}(-1) = \pi$

$\boxed{x = \pi + 2\pi N \quad N = 0, \pm1, \pm2, \dots}$

c) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$ solution 1

$\pi - \left(-\frac{\pi}{3}\right) = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$ solution 2

general solution $\boxed{\begin{array}{ll} x = -\frac{\pi}{3} + 2\pi N & N = 0, \pm1, \pm2, \dots \\ x = \frac{4\pi}{3} + 2\pi N & N = 0, \pm1, \pm2, \dots \end{array}}$

d) $2\cos^2 x = \cos x$

$2\cos^2 x - \cos x = 0$

$\cos x\,(2\cos x - 1) = 0$

$\cos x = 0$

gives $x = \pm\cos^{-1}(0) + 2\pi N$

$x = \pm\frac{\pi}{2} + 2\pi N$

$N = 0, \pm1, \pm2$

$2\cos x = 1 \quad \cos x = \frac{1}{2}$

$x = \pm\cos^{-1}\left(\frac{1}{2}\right) + 2\pi N$

$x = \pm\frac{\pi}{3} + 2\pi N$

$N = 0, \pm1, \pm2$

general solution $\boxed{\begin{array}{ll} x = \pm\frac{\pi}{2} + 2\pi N & N = 0, \pm1, \pm2, \dots \\ x = \pm\frac{\pi}{3} + 2\pi N & N = 0, \pm1, \pm2, \dots \end{array}}$

e) $2\sin^2 x + \sin x - 1 = 0$. Let $\sin x = u$

$2u^2 + u - 1 = 0$ gives

$u = \dfrac{-1 \pm \sqrt{1 - 4(2)(-1)}}{4} = \dfrac{-1 \pm 3}{4}$ $\begin{array}{l} \nearrow -1 \\ \searrow \frac{1}{2} \end{array}$

$\sin x = -1$

$\sin x = \frac{1}{2}$

$\boxed{\begin{array}{ll} x = -\frac{\pi}{2} + 2\pi N & N = 0, \pm1, \pm2, \dots \\ x = \frac{\pi}{6} + 2\pi N & N = 0, \pm1, \pm2, \dots \\ x = \frac{5\pi}{6} + 2\pi N & N = 0, \pm1, \pm2, \dots \end{array}}$

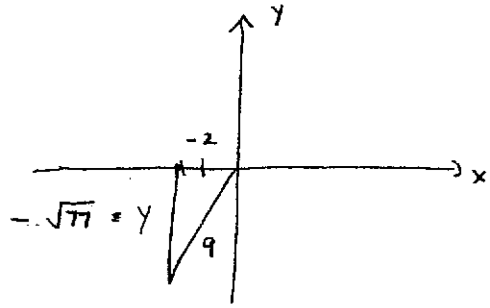
(11) $\cos(\alpha) = \frac{-2}{9} = \frac{A}{H}$

$(-2)^2 + y^2 = 9^2$

$4 + y^2 = 81$

$y^2 = 77 \quad y = -\sqrt{77}$

$\sin(\alpha) = -\frac{\sqrt{77}}{9}$

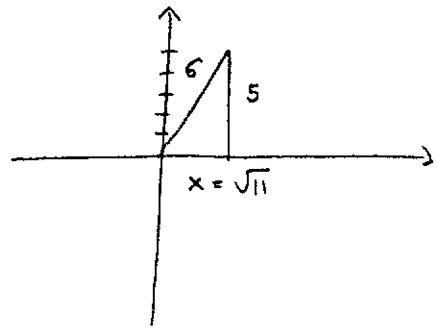


$\sin\beta = \frac{5}{6} = \frac{O}{H}$

$x^2 + 25 = 36$

$x^2 = 11 \quad x = \sqrt{11}$

$\cos\beta = \frac{\sqrt{11}}{6}$



$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta = \left(-\frac{2}{9}\right)\frac{\sqrt{11}}{6} + \left(-\frac{\sqrt{77}}{9}\right)\frac{5}{6} = \boxed{\frac{-2\sqrt{11} - 5\sqrt{77}}{54}}$