Some Content Goals for Exponential Functions

Primary content goals

- Know that functions showing exponential growth/decay have a constant of proportionality in the outputs when the inputs are consecutive.

- Represent and interpret exponential growth/decay including the *y*-intercept and the constant of proportionality in equations, input/output tables, and in graphs; interpret graphs of exponential functions without numbered scales.

- Model exponential growth/decay in equations, tables of values, and graphs.

- Compare linear change and exponential growth/decay in equations, tables, graphs, and in meaningful contexts.

Other content goals

- Calculate percent increase/decrease of a number in one step using a decimal or whole number factor.

- Relate large percent increases (+100%, +300%, etc.) to words/phrases such as “doubling”, “increase by a factor of”, and “order of magnitude”.

- Represent repeated percent increases/decreases using exponents in expressions.

- Know that multiplying a positive number by 1 does not change the number, multiplying a positive number by a number less than 1 (but larger than 0) reduces the number, and multiplying a positive number by a number greater than 1 increases the number.

Date ________________ Item _______

Always, Sometimes, or Never?

$$10 \cdot \underbrace{\_\_\_}_{\text{positive rational number}} > 10$$

Date ________________ Item _______

Maria's Safe Investment

Maria works at a university as a Laboratory Technician. She decides to start saving for her retirement by making a one-time deposit of $800 into a retirement account. This account pays guaranteed interest of 5% each year on December 31st.

How much money will be in her account after the December interest is added? Show how you figured it out.

Maria leaves her job because she enrolls in graduate school in another state. She decides to leave her retirement account in place, though, letting it grow each year.

Show how to calculate the value of her account after the December interest has been added for three years.

Date ________________ Item _______

Maria’s Riskier Investment

While she is a graduate student, Maria begins work in a new laboratory where opens a new retirement account. This time, she makes a one-time contribution of $900 into an account that does not pay a fixed, guaranteed interest rate. Instead, the value of the account may go up or down each year.

In the first year, the account loses 10%. Show how to calculate the amount in her account after the first year.

The table shows the gains or losses in Maria's riskier account for the first five years.

Show how to determine the amount in Maria's account after the five years.

Year	% Gain or Loss
1	–10%
2	+12%
3	–5%
4	+9%
5	+8%

Date _________________ Item _______

A Sales Swindle

Frederick manages a clothing store and wants to boost sales without having to reduce any prices. He tells his staff to re-mark the price of every single item in the store so that they are 50% higher. Then, Frederick puts a big sign in front of the store declaring, "*Absolutely everything is 50% off!*"

What do you think of Frederick's strategy?

Date ________________ Item _______

A Safe Investment Function

Maria's began her first retirement account with a one-time contribution of $800, and this account pays guaranteed interest of 5% each year on December 31st.

Complete the first three rows of the table. In the middle column, show the expression that can be used to determine the amount in the account after each year. Use an exponent if there is repeated multiplication. In the right-hand column, determine the dollar amount that will be in the account after that many years have elapsed.

Years elapsed	An expression showing how to calculate the amount in the account after this many years	Value of the account after this many years
2		
3		
4		

Date ________________ Item _______

Gambling Away Her Retirement Account

Maria retires after a full career of research and teaching. She suddenly remembers she has an account from her first lab job that she started with just $800, and that has been earning 5% interest every year for 40 years. Maria takes the money out of the retirement account and pays taxes on it, depositing the $5,000 that is left in a new bank account she will use for her trips to the casino.

Every week before boarding the free bus to the casino, Maria withdraws 10% of her account and loses it on the slot machines.

1. Write a function that can be used to show how much money remains is in her bank account (*y*) after *x* weeks visiting the casino.

2. Complete the table.

Weeks	An expression showing how to calculate the amount in the account after this many weeks	Value of the account after this many weeks
0		
1		
2		
3		
4		
5		
10		

3. Will Maria still have any money left to visit the casino after a year of weekly trips? If so, how much?

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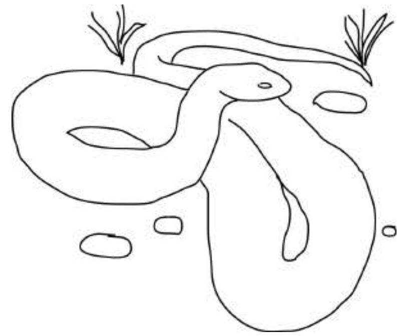
Working with Exponential Functions



1. The Lake Erie water snake can be found near the shores of islands that lie in Lake Erie. It may bite humans if provoked, but it is not venomous.

   Because of the homes and tourism on the islands, the Lake Erie water snake population plummeted because of killings and their loss of habitat. As a result, the water snake was listed as threatened under the Endangered Species Act.

   An adult population of 5,450 adult water snakes was measured, and then over the next six years as a variety of conservation efforts were enacted on the islands to educate the public about the importance of the water snakes. The following function gives a pretty good estimate of how the snake population rebounded during those six years:

$$y = 5450(1.135)^x$$

   where *x* represents the number of years since conservation efforts began, and *y* represents the adult Lake Erie water snake population.

   a. What is the significance of the number 1.135 in the function?

   ____

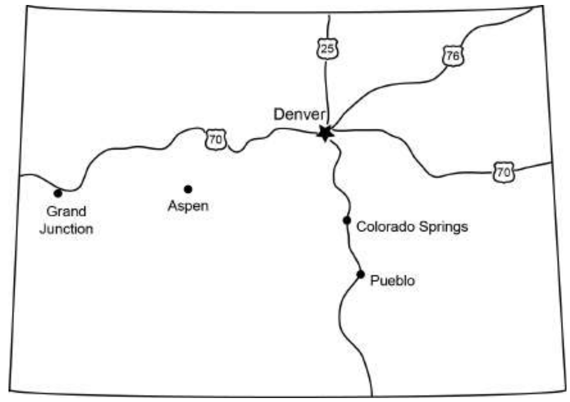
   ____

   ____

   b. Determine the water snake population after 6 years of conservation efforts. Show how you figured it out.

   c. The goal of a group of conservationists is for the water snake population to reach 25,000. If the growth rate shown in the function continues, how many years should it take to reach the goal population?

2. State officials need to estimate population changes in order to plan long-term budgets for vital infrastructure such as roads, schools, hospitals, electrical power, and other community needs. In 2000, the population of Colorado was approximately 3,700,000. At that time, statisticians predicted that state population growth would be 2% per year.



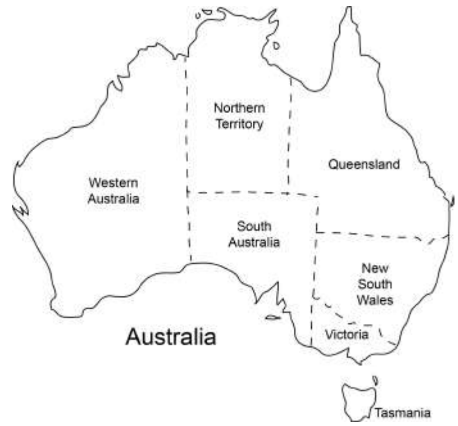
a. Write an exponential function that the statisticians could have used to estimate the Colorado population in the years after 2000. Use *x* to represent the number of years after 2000, and *y* to represent the estimated population after that many years.

a. Explain what the numbers in your function represent.

b. Show how the statisticians could have used this function to predict the Colorado population in 2017.

c. Using a smartphone or nearby computer, search for the actual population in Colorado in 2017. How did the function estimate compare with the reality? What consequences could have resulted from any difference between the two?

3. Rabbits were first brought to Australia in 1788 when English ships sailed to the continent to establish a penal colony. This voyage was known as the "First Fleet", but these were not the first humans to set foot on what would become Australia. Indigenous peoples had already lived there for more than 40,000 years.



The ships brought rabbits that would be held in cages and used as food animals for the colony. The rabbit population was well controlled over the next 70 years.

In the 1850s, the colonist Thomas Austin missed the hunting he used to do. He had his nephew send him twelve rabbits from England. Austin added to this group another dozen local rabbits that were being raised for food, and in 1859 he released all 24 on his property so he could hunt them.

The wild rabbit population grew at a tremendous rate after this release until the 1920s. A function can be used to estimate the population of wild rabbits in Australia after the release:

$$y = 24(1.385)^x$$

where x represents the number of years elapsed after the rabbit release, and y represents the approximate wild rabbit population after that many years.

What is the yearly change in the wild rabbit population indicated by the function? ___________

Complete the table for the function.

Actual Year	Adjusted Year	An expression showing how to calculate the growth in rabbits	Estimated Australian rabbit population
1859	0		24
1860	1		
1861	2		
1899			
1920			

The effect of the rabbit explosion on the ecology of Australia was devastating. See the Wikipedia page titled "*Rabbits in Australia*" for more information on the damage rabbits have caused there, and the efforts that have been undertaken over many decades to control their growth.

4. For each of the following functions, first identify whether the function shows exponential growth or decay. Then, determine the y-intercept, and finally the percent increase/decrease in the outputs when the inputs increase consecutively.

$y = 4(.9)^x$

Exponential growth or decay? ___________________

y-intercept: _________

Percent increase/decrease: ___________________

$y = (1.5)^x$

Exponential growth or decay? ___________________

y-intercept: _________

Percent increase/decrease: ___________________

$y = 6\left(\frac{3}{4}\right)^x$

Exponential growth or decay? ___________________

y-intercept: _________

Percent increase/decrease: ___________________

$y = 8(1)^x$

Exponential growth or decay? ___________________

y-intercept: _________

Percent increase/decrease: ___________________

$y = 0.8(1.005)^x$

Exponential growth or decay? ___________________

y-intercept: _________

Percent increase/decrease: ___________________

$y = 3(2)^x$

Exponential growth or decay? ___________________

y-intercept: _________

Percent increase/decrease: __________________

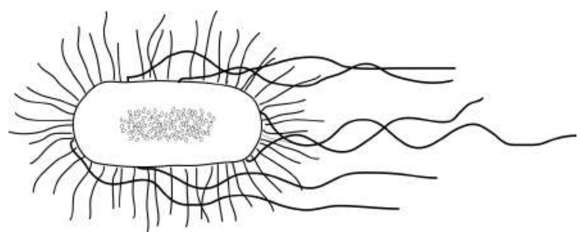
Date _________________ Item _______

Representing Large Percentage Changes

$$y = 3(2)^x$$

x	0	1	2	3	4	5
y						

E. coli (pictured at right) is a bacteria that lives in our intestines, where it helps our bodies break down and digest the food we eat. Most E. coli are harmless, and are actually an important part of a healthy person's digestive system. But some types of E. coli can cause illness, either diarrhea or an illness outside of the digestive system.



E. coli bacteria have been studied extensively in laboratories. When they are grown in a lab in optimum conditions (where it is warm and where there are plenty of nutrients), E. coli can grow by a factor of eight every hour.

Write an expression showing how to calculate the number of E. coli bacteria that will be present when a starting number of 50 bacteria are given optimum growing conditions for 3 hours.

In rural areas of the U.S., the number of children who have been born with drug dependencies has increased by a factor of six since 2004.[1]

The number of women held in local U.S. jails is 14 times larger than it was in 1970.[2]

The number of U.S. anti-Muslim hate groups has tripled this year.[3]

Climate change combined with increased urbanization around the world has meant that when we compare the global annual cost of natural disasters today versus 30 years ago, the annual cost has risen by an order of magnitude.[4]

Since 2010, there has been a five-fold increase in the global number of refugee and migrant children moving alone.[5]

The first USB flash drive appeared on the market in 2000 and had a storage capacity of 8 megabytes. Flash drives were developed over the next several years that held larger and larger amounts of storage. In 2017, a 2 terabyte flash drive was announced, which is equal to 2,000,000 megabytes.

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[1] *Harper's Index*, March 2017.
[2] *Harper's Index*, December 2016.
[3] *Harper's Index*, May 2017.
[4] "*The Making of a Riskier Future: How Our Decisions Are Shaping Future Disaster Risk*", a 2016 report by the Global Facility for Disaster Reduction and Recovery.
[5] *"A Child is a Child: Protecting Children on the Move From Violence, Abuse, and Exploitation"*, a 2017 report by UNICEF.

Date ________________ Item _______

A High Risk—High Reward Opportunity

A friend of yours lives in a nearby country where a drug company is looking for volunteers in a risky experiment. Volunteers will be paid a great deal of money to take a daily pill for 14 days. The down side is that the drug has potentially terrible side effects.

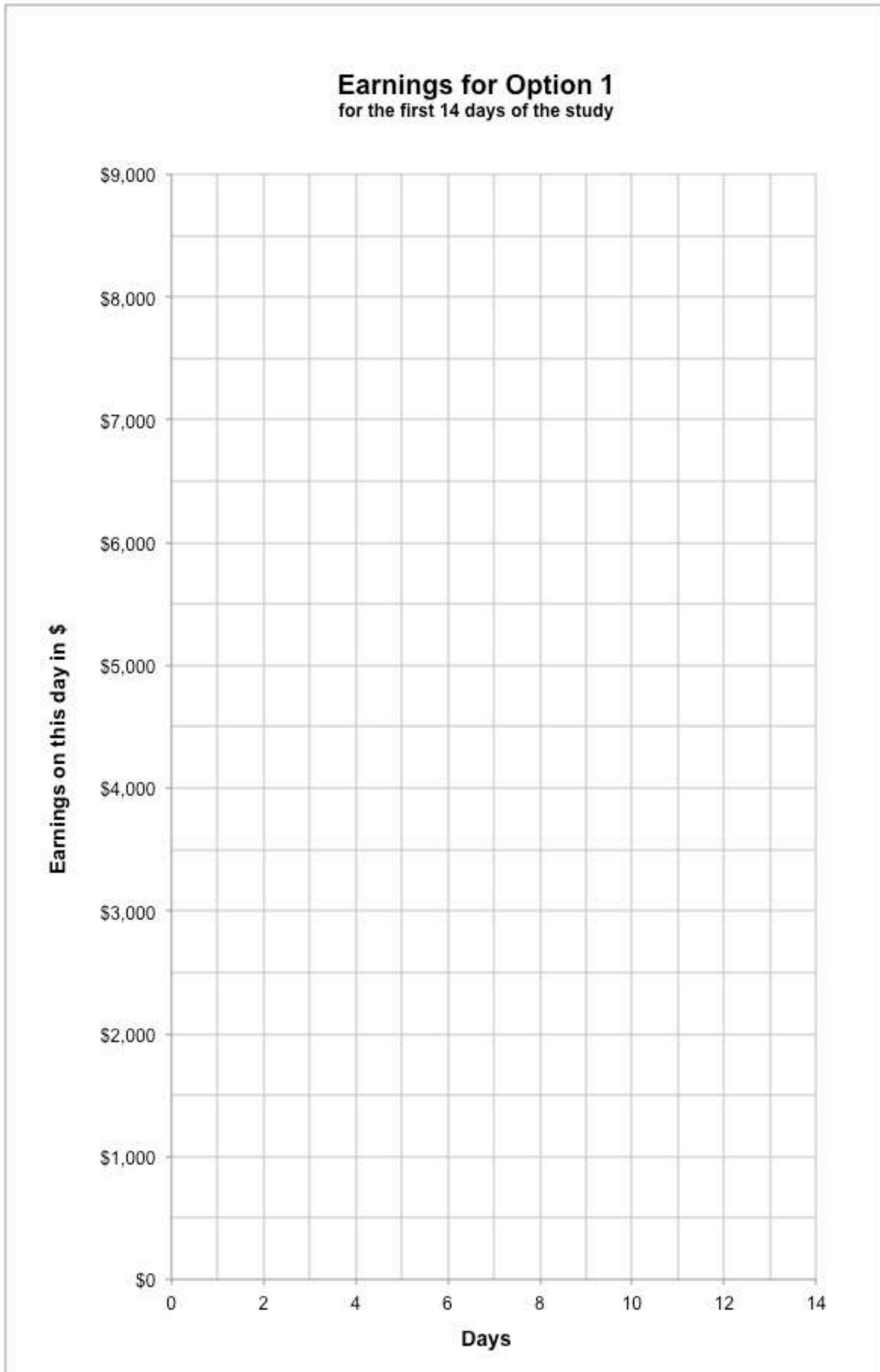
Despite the risks, your friend has decided to participate in the experiment. The company has offered her two payment options:

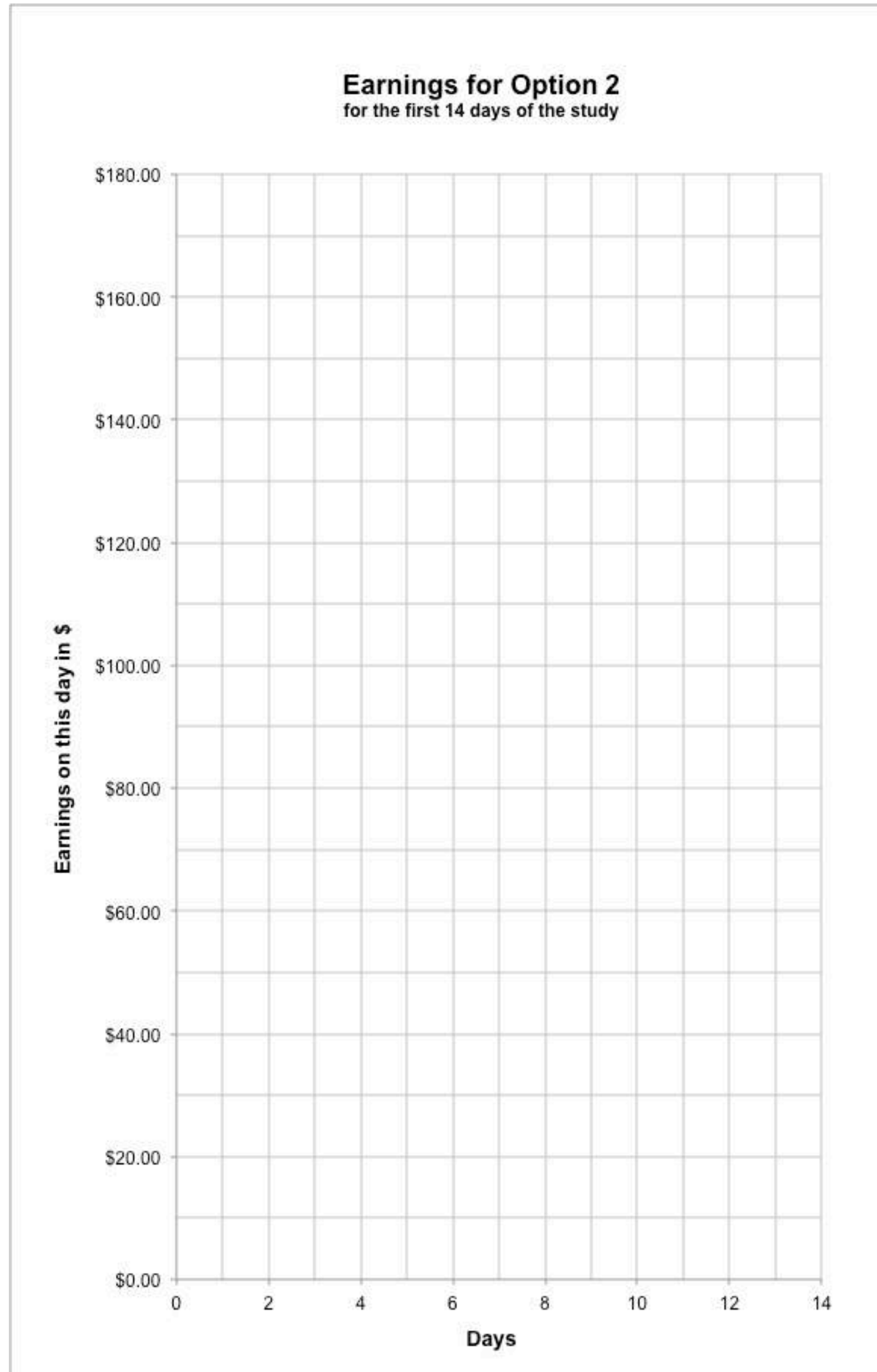
Option 1: She will earn $1,000 just for signing up, and then she earns $500 more than that each day she takes the pill. That means $1,500 for the first day, $2,000 for the second day, etc.

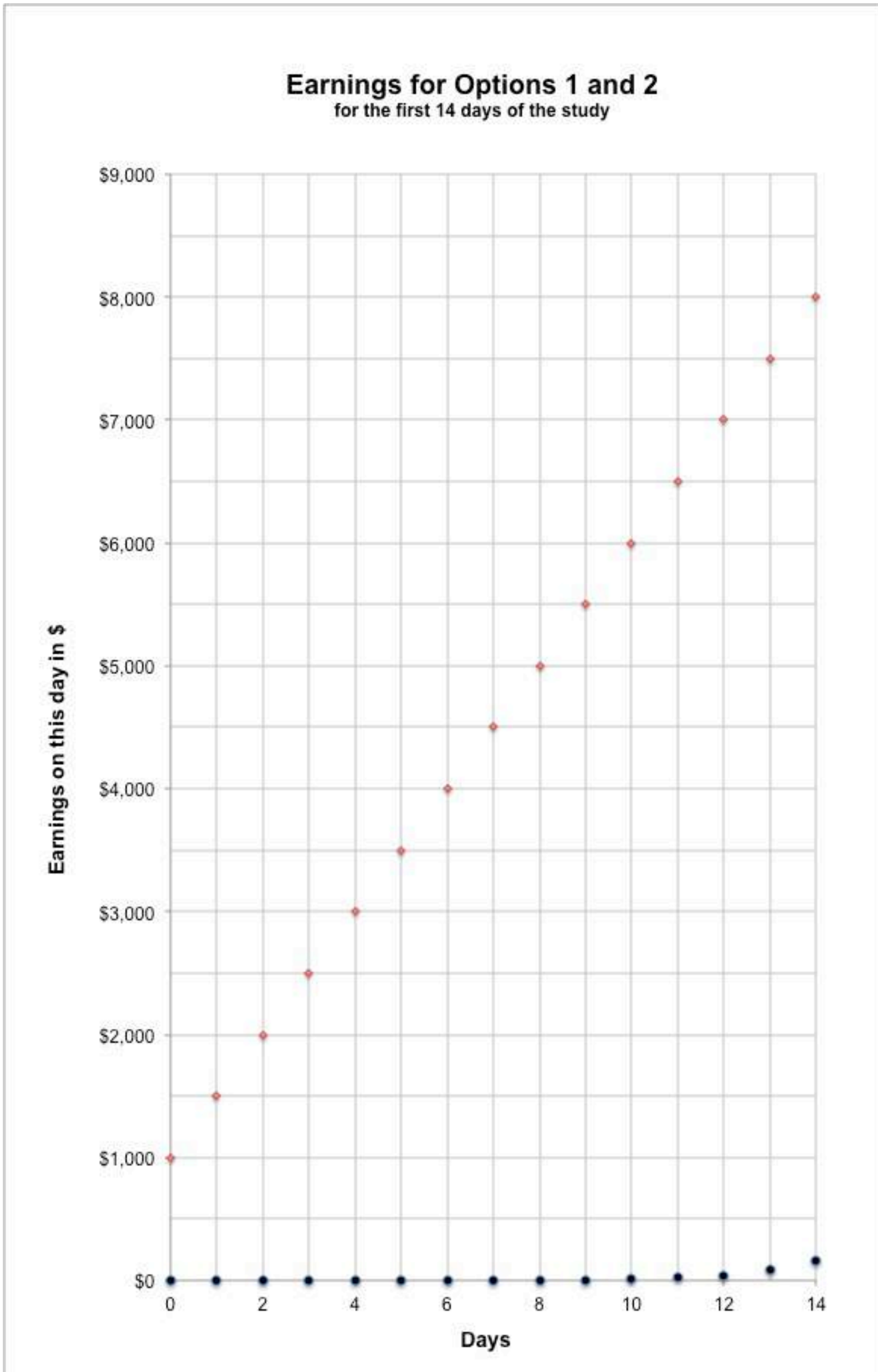
Option 2: She will earn one penny just for signing up, and that amount will double each day she takes the pill. That means $0.02 for the first day, $0.04 for the second day, etc.

Function for Option 1:	
Day	Amount she earns on this day
0	$1,000
1	$1,500
2	$2,000
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	

Function for Option 2:	
Day	Amount she earns on this day
0	$0.01
1	$0.02
2	$0.04
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	


Earnings for Option 1
for the first 14 days of the study
$9,000
$8,000
$7,000
$6,000
$5,000
$4,000
$3,000
$2,000
$1,000
$0
0
2
4
6
8
10
12
14
Days
Earnings on this day in $


Earnings for Option 2
for the first 14 days of the study
$180.00
$160.00
$140.00
$120.00
$100.00
$80.00
$60.00
$40.00
$20.00
$0.00
0
2
4
6
8
10
12
14
Days
Earnings on this day in $


Earnings for Options 1 and 2
for the first 14 days of the study
$9,000
$8,000
$7,000
$6,000
$5,000
$4,000
$3,000
$2,000
$1,000
$0
0
2
4
6
8
10
12
14
Days
Earnings on this day in $

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Identifying Linear and Exponential Change in Function Tables

For each of the following input/output tables, determine whether the values could be represented by a linear function, an exponential function, or neither. Record the important features of the functions that you identify (slope or constant of proportionality (COP), and *y*-intercept).

x	y
0	10
1	15
2	20
3	25

Linear or Exponential? ______________

Slope or COP (circle one): _____

y-intercept: _____

function equation: ___________________

x	y
0	10
1	30
2	90
3	270

Linear or Exponential? ______________

Slope or COP (circle one): _____

y-intercept: _____

function equation: ___________________

x	y
0	128
1	64
2	32
3	16

Linear or Exponential? ______________

Slope or COP (circle one): _____

y-intercept: _____

function equation: ___________________

x	$f(x)$
0	12
2	9
4	6
6	3

Linear or Exponential? ______________

Slope or COP (circle one): _____

y-intercept: _____

function equation: ___________________

x	y
–1	16
0	8
1	4
2	2

Linear or Exponential? ______________

Slope or COP (circle one): _____

y-intercept: _____

function equation: ___________________

x	y
–2	5
0	5
2	5
4	5

Linear or Exponential? ______________

Slope or COP (circle one): _____

y-intercept: _____

function equation: ___________________

x	y
–2	4
–1	1
0	0
1	1

Linear or Exponential? ______________

Slope or COP (circle one): _____

y-intercept: _____

function equation: ___________________

x	y
–2	$\frac{1}{4}$
–1	$\frac{1}{2}$
0	1
1	2

Linear or Exponential? ______________

Slope or COP (circle one): _____

y-intercept: _____

function equation: ___________________

x	y
2	10
3	5
4	0
5	–5

Linear or Exponential? ______________

Slope or COP (circle one): _____

y-intercept: _____

function equation: ___________________

x	y
0	4
1	6
2	9
3	13.5

Linear or Exponential? ______________

Slope or COP (circle one): _____

y-intercept: _____

function equation: ___________________

Date _________________ Item _______

Hungry Hungry Minnows

A river has an initial minnow population of 40,000 that is growing at 5% per year. The minnows rely on algae in the river for food. At the time that the minnow population was measured at 40,000, there is enough algae to support 55,000 minnows.

Due to environmental conditions, the amount of algae that the minnows use for food is decreasing. This is shown in the table.

Complete the table using the above information to predict the changes in the minnow population for the first three years.

Years elapsed	An expression showing how to calculate the number of minnows after this many years	Number of minnows after this many years	Number of minnows that can be supported by algae
0			55,000
1			54,000
2			53,000
3			52,000

1. Is the change in the minnow population for the first three years exponential, linear, or some other kind of change? How do you know?

2. Create a function that models the predicted minnow population for the first three years where *x* represents the number of years elapsed, and *y* represents the minnow population after that many years.

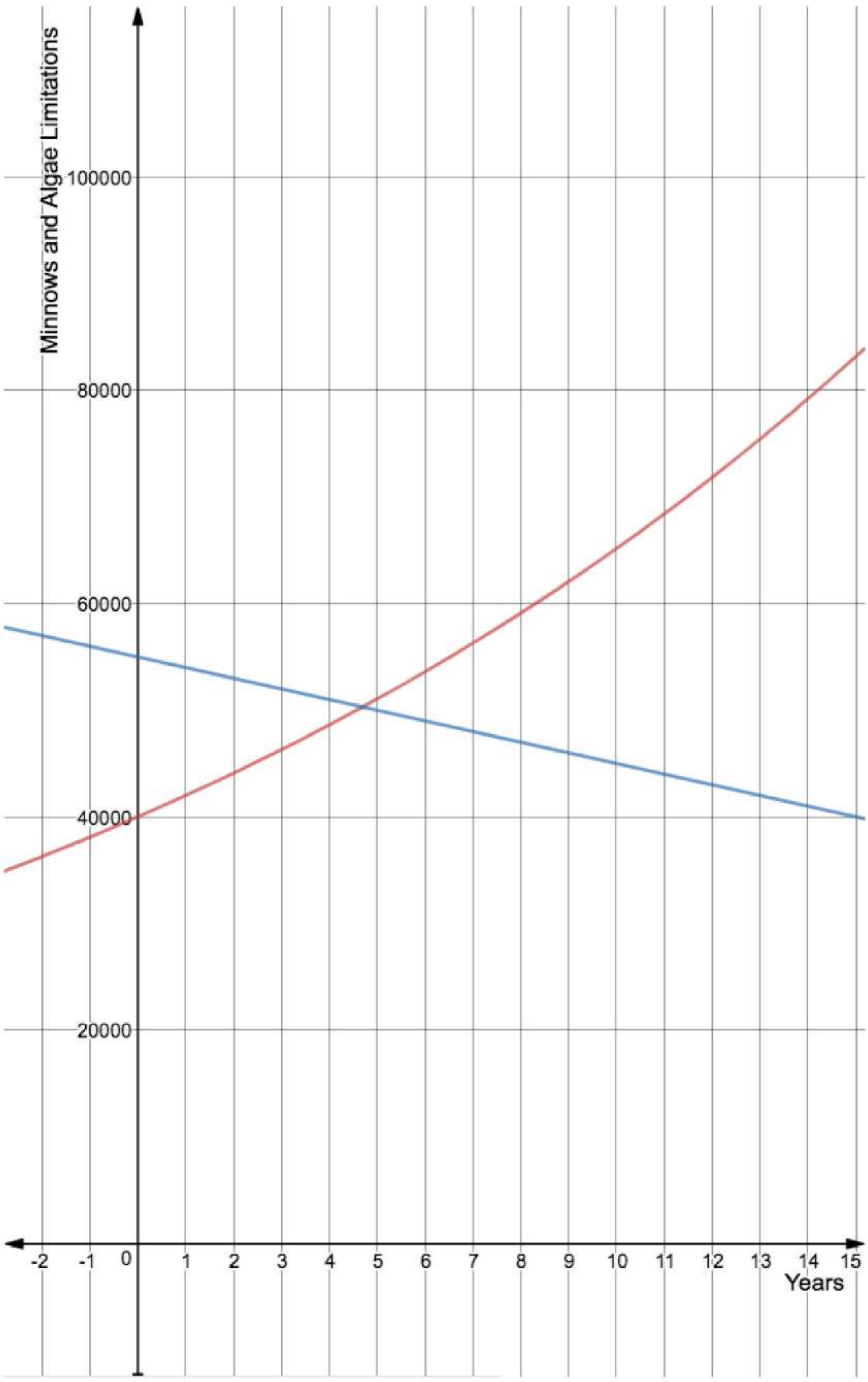
3. Explain what the numbers in your function represent.

4. Is the change in the number of minnows that the algae can support exponential, linear, or some other kind of change? How do you know?

5. Create a function that models the predicted decrease in the algae where *x* represents the number of years elapsed, and *y* represents the minnow population that can be supported by algae after that many years.

6. Explain what the numbers in your function represent.

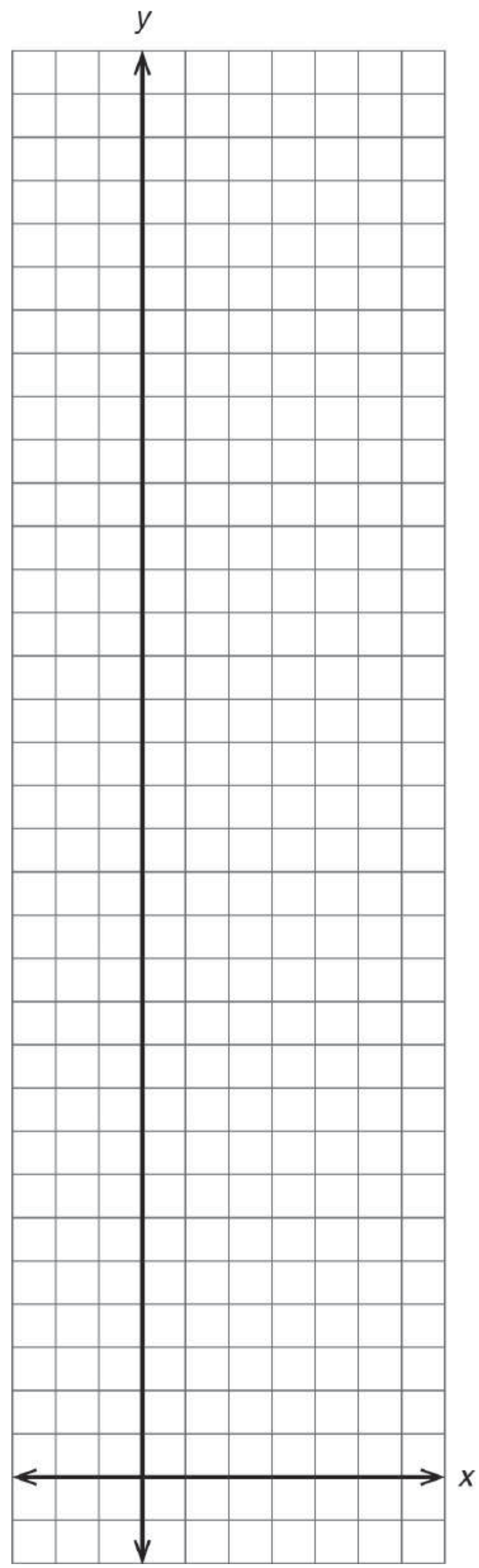
7. What is going to happen to the minnows? When?


Minnows and Algae Limitations
100000
80000
60000
40000
20000
0
-2
-1
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
Years

Date ________________ Item _______

Graphing Exponential Functions in Multiple Quadrants, introduction

$y = 8(.5)^x$	
x	y
–2	
–1	
0	
1	
2	
3	
4	
5	
6	



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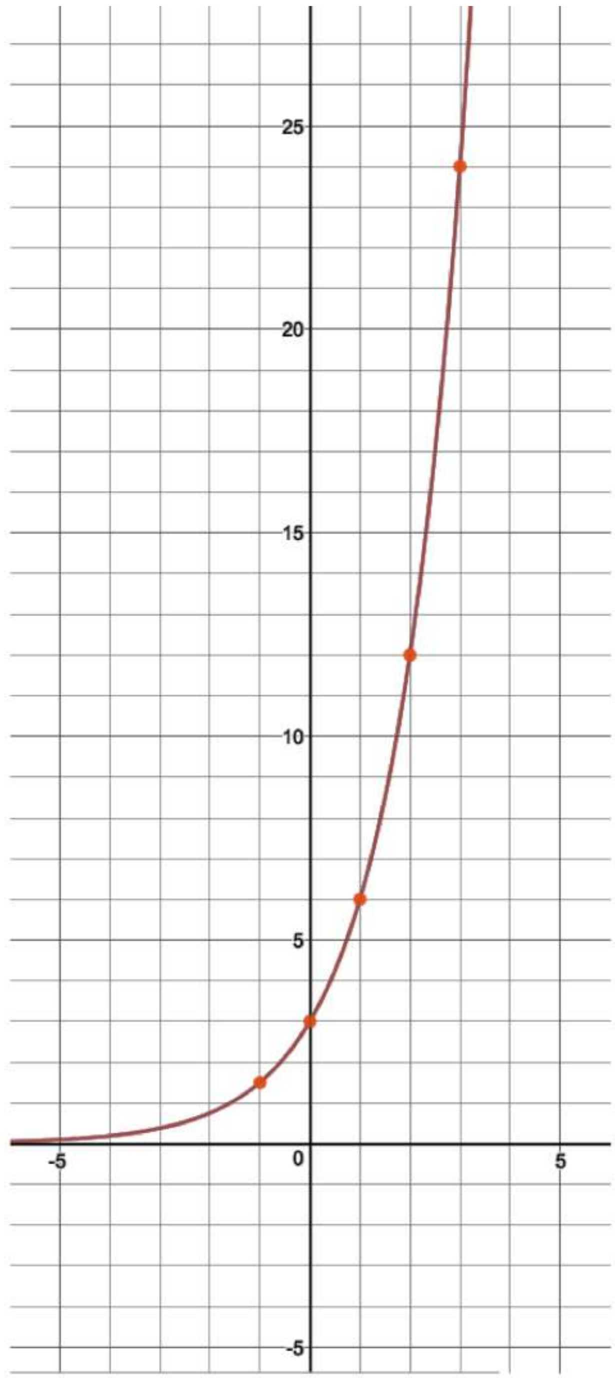
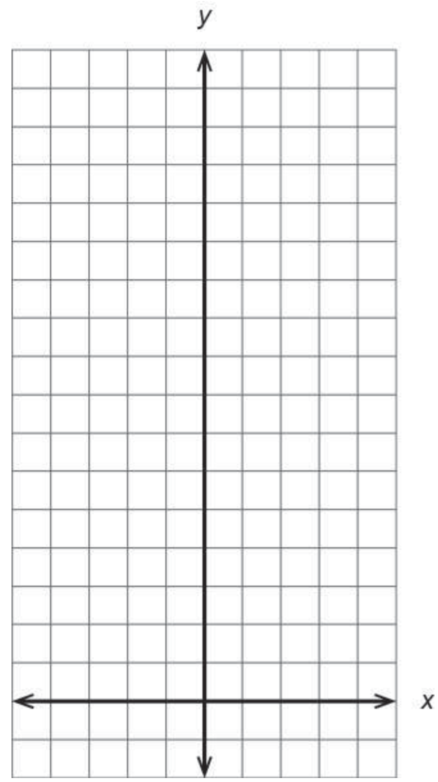
Three Views of Exponential Functions, practice

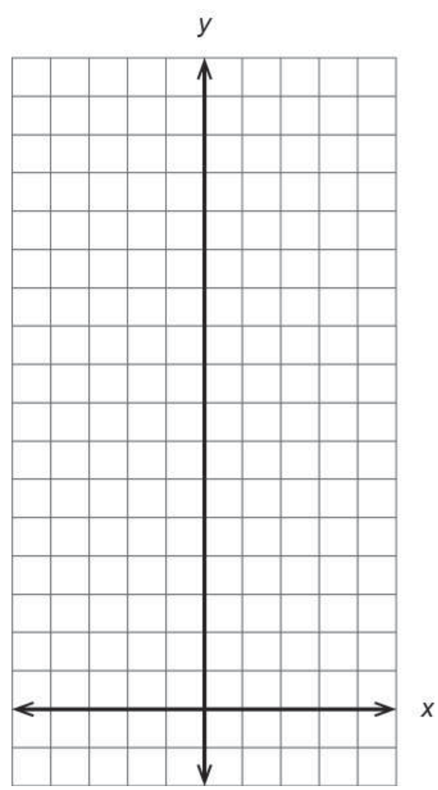
Function:	
x	*y*
–1	
0	
1	
2	
3	
4	
5	



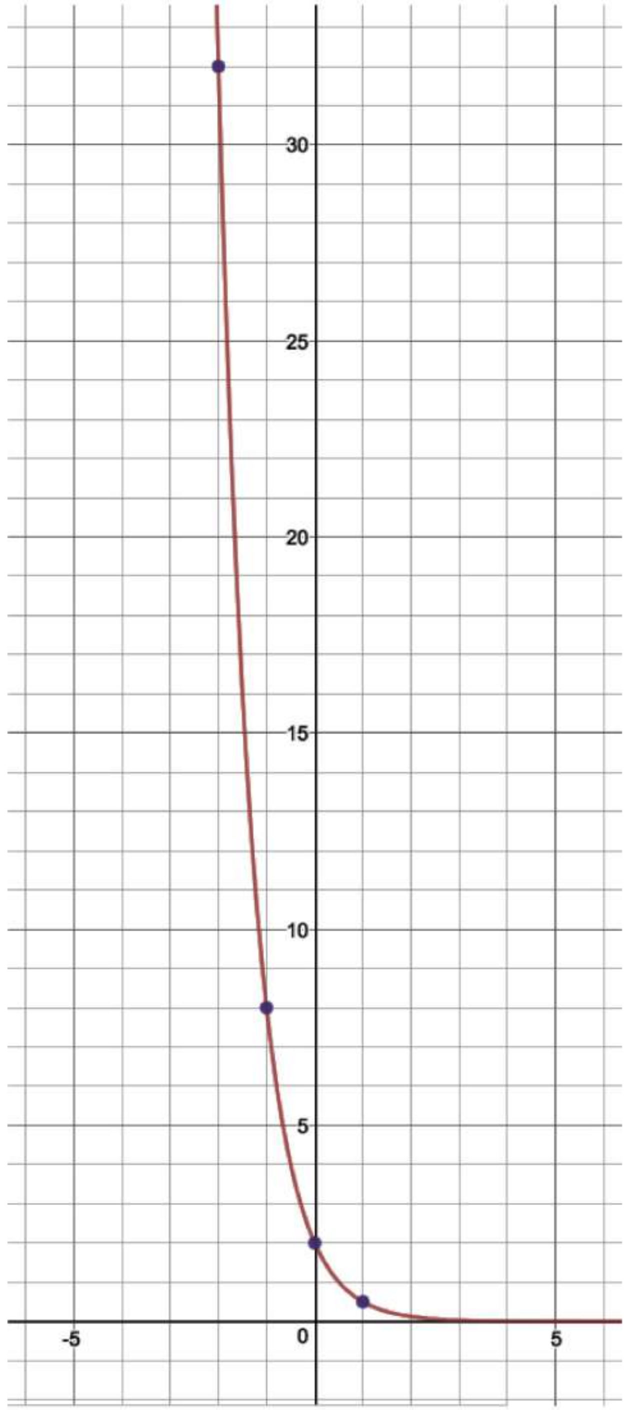
$y = 2^x$	
x	y
–3	
–2	
–1	
0	
1	
2	
3	
4	



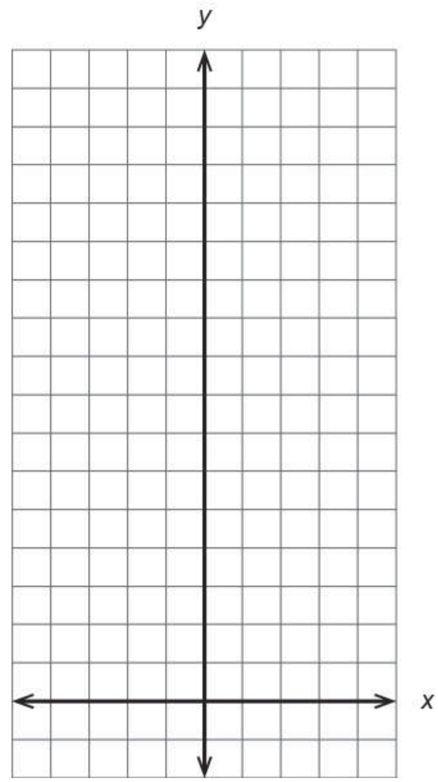
Function:	
x	y
–3	$\frac{1}{27}$
–2	$\frac{1}{9}$
–1	$\frac{1}{3}$
0	1
1	3
2	9
3	27



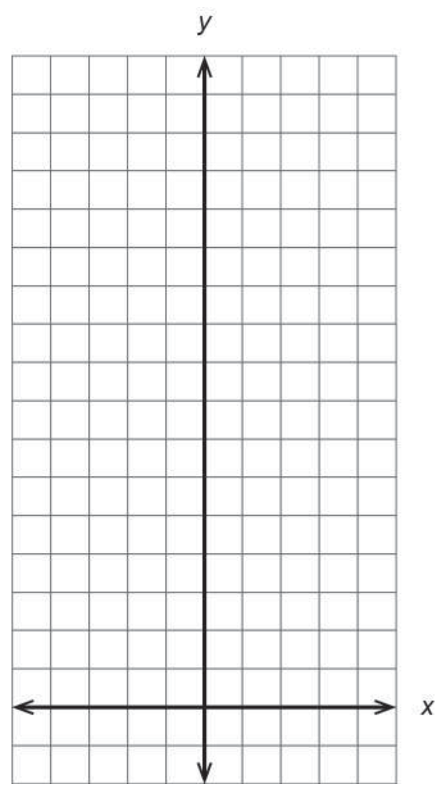
Function:	
x	*y*
–2	
–1	
0	
1	$\frac{1}{2}$
–6	
5	



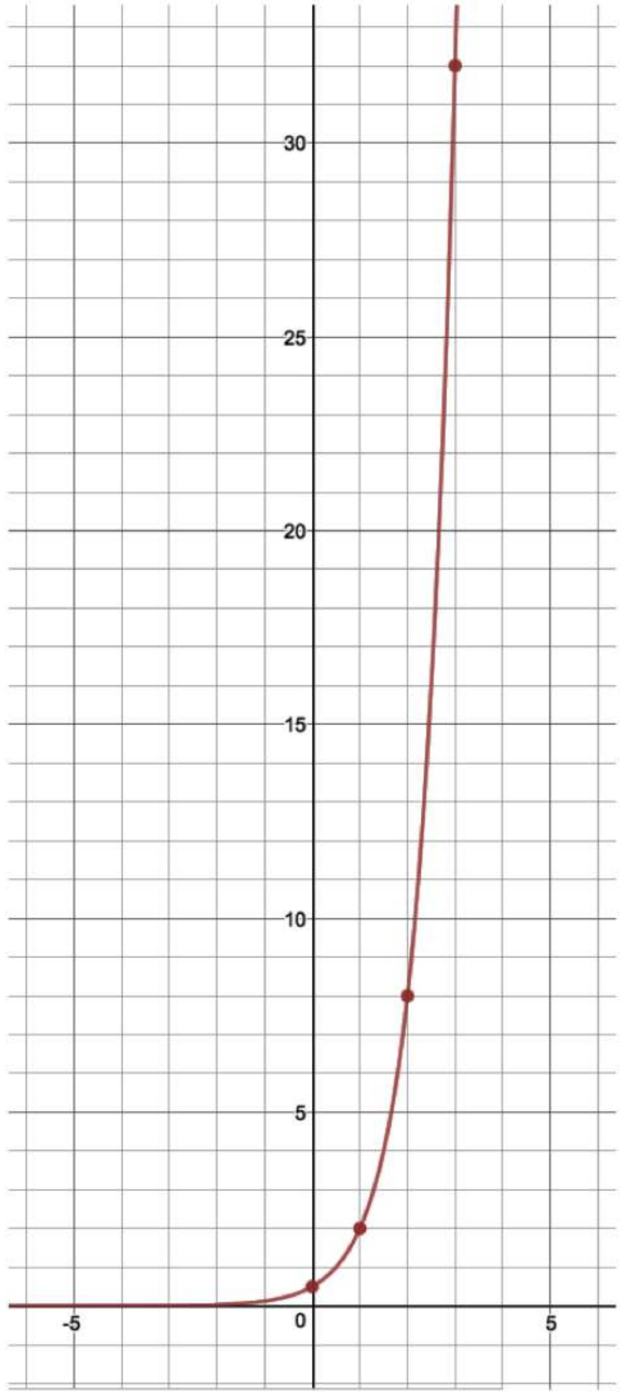
$y = .5(2)^x$	
x	y
5	
4	
3	
2	
1	
0	
–1	
–2	



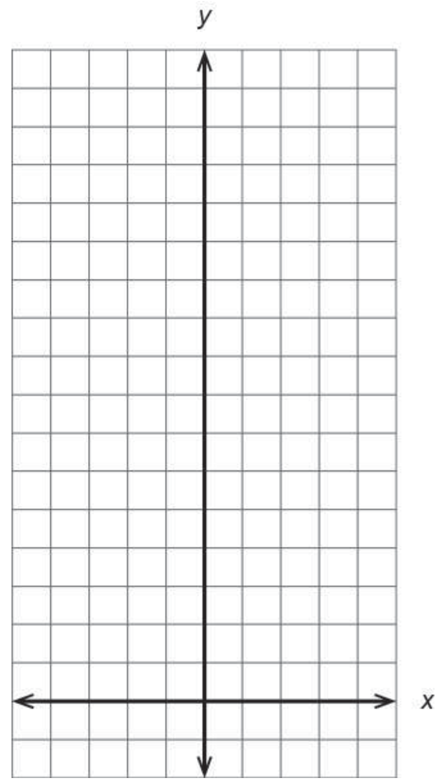
Function:	
x	y
–2	16
–1	8
0	4
1	2
2	1
3	$\frac{1}{2}$
10	
–5	



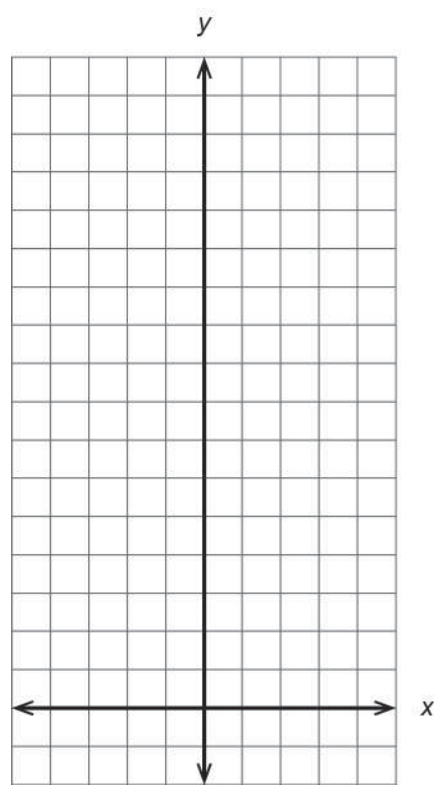
Function:	
x	y
0	$\frac{1}{2}$
1	
2	
3	
–2	
6	



$y = .5(4)^x$	
x	y
–1	
0	
1	
2	
3	
–5	
8	



Function:	
x	y
–1	$\frac{1}{6}$
0	$\frac{1}{2}$
1	$\frac{3}{2}$
1	$\frac{9}{2}$
2	$\frac{27}{2}$



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Exponential Function Graphs Without Numbered Scales

1. The graph shows an exponential function. Which of the following could be an equation for the function?

   a. $f(x) = 2(3)^x$

   b. $y = 0.5\left(2\right)^x$

   c. $y = 2x + 3$

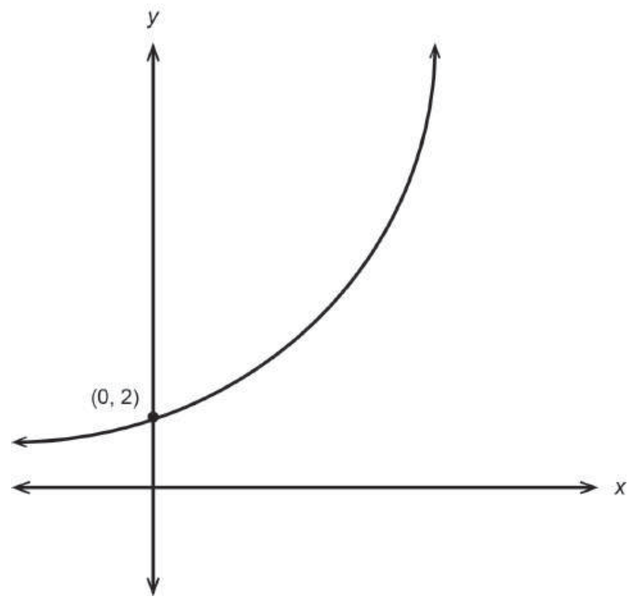
   d. $g(x) = 2(.5)^x$

   e. $h(x) = x^2 + 2$

   Explain how you decided your answer.

   ____

   ____



2. The graph shows an exponential function. Which of the following could be an equation for the function?

   a. $y = 3(2)^x$

   b. $f(x) = 16(.5)^x$

   c. $y = -2x + 8$

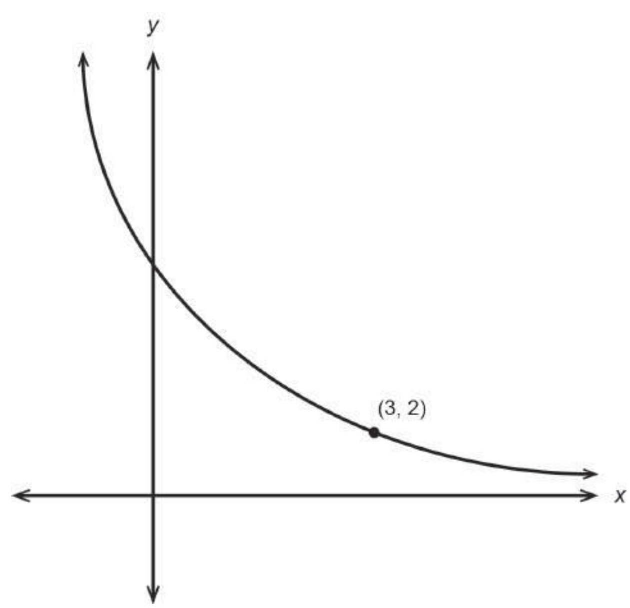
   d. $y = 16\left(\frac{1}{2}\right)^x$

   e. $y = 2^x$

   Explain how you decided your answer.

   ____

   ____



3. The graph shows an exponential function. Which of the following could be an equation for the function?

   a. $h(x) = 3(8)^x$

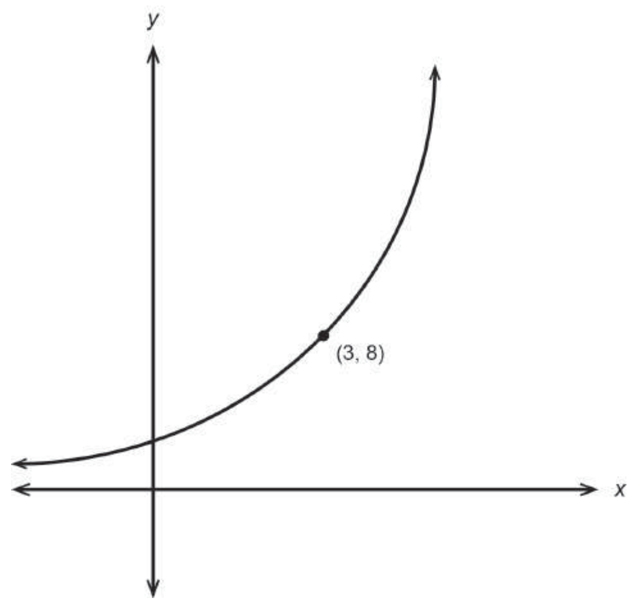
   b. $f(x) = .5(2)^x$

   c. $y = 2^x$

   d. $y = .25(4)^x$

   e. $g(x) = 64(.5)^x$

   Explain how you decided your answer.



4. The graph shows an exponential function. Which of the following could be an equation for the function?

   a. $h(x) = 2(4)^x$

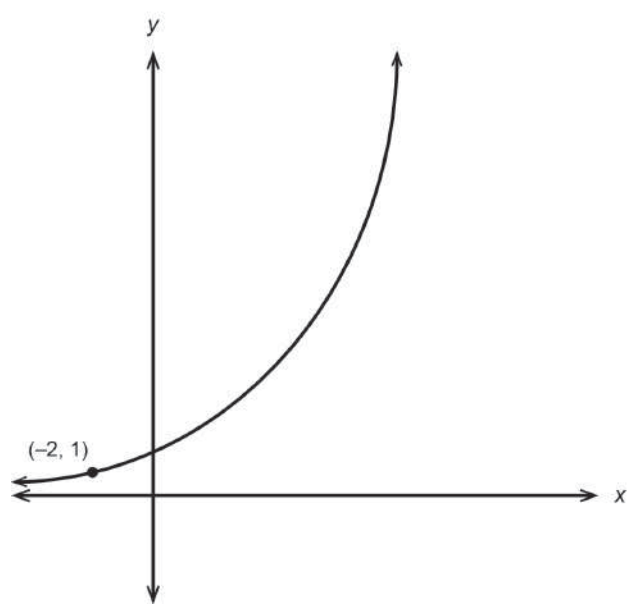
   b. $y = 4(2)^x$

   c. $y = 4\left(\frac{1}{2}\right)^x$

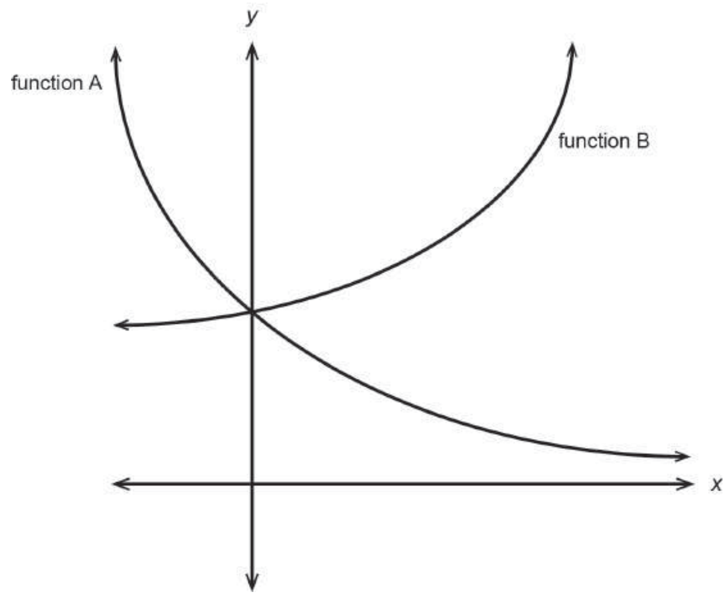
   d. $y = .5(2)^x$

   e. $y = 16^x$

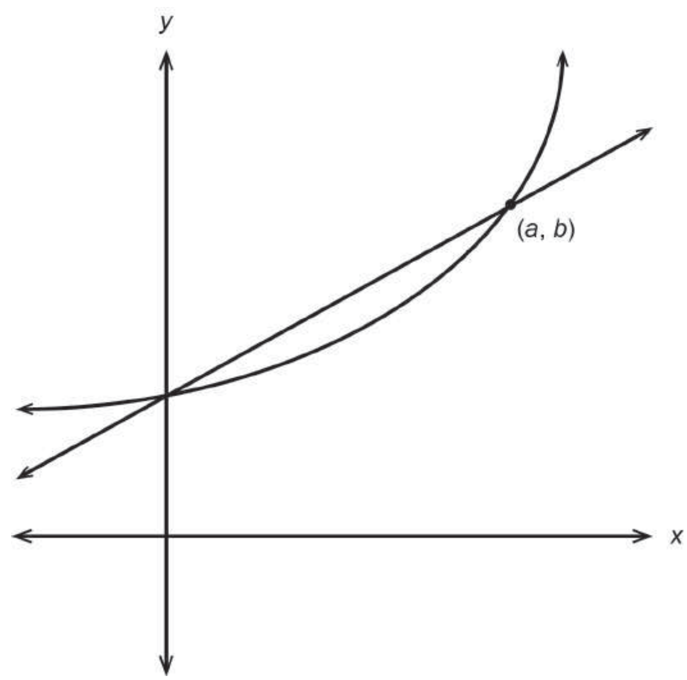
   Explain how you decided your answer.



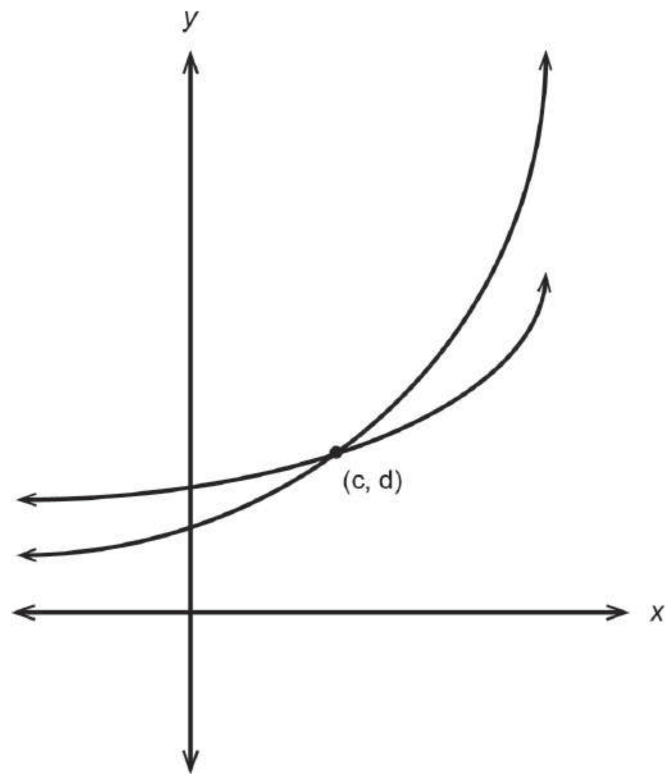
5. The graph shows two exponential functions. What can you determine, if anything, about these functions?



6. The graph shows a linear function and an exponential function. What can you determine, if anything, about these functions? And what, if anything, do you know about the coordinates of the point (*a*, *b*)?



Challenge



The figure shows two exponential functions that intersect at the point (c, d). Here are the function equations:

$$f(x) = a(3)^x \qquad g(x) = b(2)^x$$

We have hidden numbers in the function equations by replacing them with *a* and *b*.

On the graph, can you determine which function is which?

Is it possible to determine which is a larger number, *a* or *b*?