Key - EXAM 3 Review

① gcd $= 3^5 * 5^3 * 13$

② No, $\gcd(14, 98) = 14 \neq 1$

③
$54321 = 12345\,(4) + 4941$ $\quad (12345 * 4 = 49380)$

$12345 = 4941\,(2) + 2463$ $\quad (4941 * 2 = 9882)$

$4941 = 2463\,(2) + 15$ $\quad (2463 * 2 = 4926)$

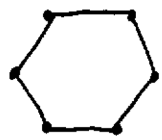
$2463 = 15\,(164) + \boxed{3}$ $\quad (15 * 164 = 2460)$

$15 = 3\,(5) + 0$

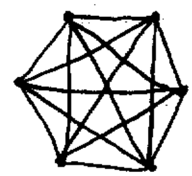
$\gcd(12345, 54321) = 3$ (the last nonzero remainder)

④

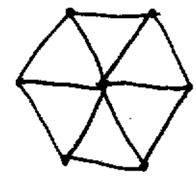
$K_{3,5}$



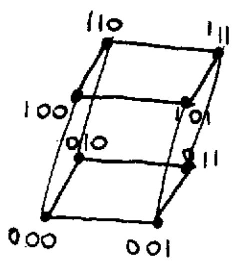
C_6



K_6



W_6



⑤ 4 1 7 use table

100 001 111

$\boxed{(417)_8 = (100001111)_2}$

⑥

$417 = 2\,(208) + 1$

$208 = 2\,(104) + 0$

$104 = 2\,(52) + 0$

$52 = 2\,(26) + 0$

$26 = 2\,(13) + 0$

$13 = 2\,(6) + 1$

$6 = 2\,(3) + 0$

$3 = 2\,(1) + 1$

$1 = 2\,(0) + 1$ stop at quotient 0

$\boxed{417 = (110100001)_2}$

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⑦ 0010 | 1111 | 0111 use table

2 F 7

$(2F7)_{16}$

⑧ $(1011110111)_2 = 1 * 2^9 + 1 * 2^7 + 1 * 2^6 + 1 * 2^5 + 1 * 2^4 + 1 * 2^2 + 1 * 2 + 1 = \boxed{759}$

(9) Let $P(n)$ be the statement

$1*2 + 2*3 + \ldots + n(n+1) = \dfrac{n(n+1)(n+2)}{3}$ for the positive integer n

BASIS STEP $P(1)$ is the statement $1*2 = \dfrac{1(1+1)(1+2)}{3}$

Since both sides of $P(1)$ are equal (to 2), $P(1)$ is true.

INDUCTIVE STEP $P(K)$ is the statement (inductive hypothesis)

$1*2 + 2*3 + \ldots + K(K+1) = \dfrac{K(K+1)(K+2)}{3}$

$P(K+1)$ is the statement $1*2 + 2*3 + \cdots + K(K+1) + (K+1)(K+2) = \dfrac{(K+1)(K+2)(K+3)}{3}$

We prove that $P(K)$ implies $P(K+1)$

$$\boxed{1*2 + 2*3 + \cdots + K(K+1)} + (K+1)(K+2) \underset{\text{by I.H.}}{=} \boxed{\frac{K(K+1)(K+2)}{3}} + (K+1)(K+2) \underset{\text{algebra}}{=}$$

$$\frac{K(K+1)(K+2) + 3(K+1)(K+2)}{3} = \frac{(K+1)(K+2)(K+3)}{3}$$, as desired.

By Mathematical Induction, $P(n)$ is true for every positive integer n.

(10)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

M A T H

12 0 19 7

f ↓ ↓ ↓ ↓

9 7 8 6

↓ ↓ ↓ ↓

J H I G

$f(12) = (11*12 + 7) \text{ MOD } 26 = 139 \text{ MOD } 26 = 9$

$f(0) = 7 \text{ MOD } 26 = 7$

$f(19) = (11*19 + 7) \text{ MOD } 26 = 216 \text{ MOD } 26 = 8$

$f(7) = (11*7 + 7) \text{ MOD } 26 = 84 \text{ MOD } 26 = 6$

$\boxed{\text{JHIG}}$

(11) P B S O X N

15 1 18 14 23 13

f^{-1} ↓ ↓ ↓ ↓ ↓ ↓

5 17 8 4 13 3

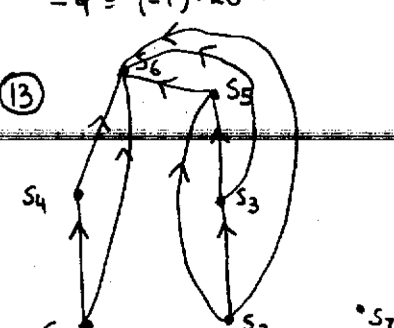
$\boxed{\text{F R I E N D}}$

$f^{-1}(p) = (p-10) \bmod 26$

$f^{-1}(1) = -9 \bmod 26 = 17$

$-9 = (-1)\cdot 26 + 17$

(13)



$s = 5$
$t = 8$
$t = 9$
$u = 5$
$t = 11$
$s = 16$

(12) $f(0) = -1 \quad f(1) = 2$

$f(2) = 3f(1)^2 - 4(f(0))^2 = 3\cdot 4 - 4(-1)^2 = 12 - 4 = \boxed{8}$

$f(3) = 3f(2)^2 - 4f(1)^2 = 3\cdot 64 - 4\cdot 4 = 192 - 16 = \boxed{176}$

$f(4) = 3f(3)^2 - 4f(2)^2 = 3(176)^2 - 4(64) = \boxed{92{,}672}$

(14) a) It is not strongly connected, because there is no path starting at c

It is weakly connected because the graph without directions is connected

b) Yes, it is a path

It is not a circuit, it is not simple ({b,a} is a multiple edge)

length is 7