(2)

$$f: \begin{array}{l} a \to b \\ b \to a \\ c \to d \\ d \to b \end{array}$$

a) f is not one to one because 2 different inputs have the same output, $f(a) = b$ $f(d) = b$

b) f is not onto because the codomain is $\{a,b,d\}$ and the range is $\{a,b,c,d\}$. So codomain $\neq$ range

(3) $f: \mathbb{Z} \to \mathbb{Z}$, $n \to 4n$

a) f is one to one. Let $f(a) = f(b)$ where a and b are integers. Then $4a = 4b$, so $a = b$.

b) f is not onto. The range is the set of multiples of 4, which is different from the codomain $\mathbb{Z}$

(or: $1 \in$ codomain, but $1 \notin$ range since 1 is not a multiple of 4, so codomain $\neq$ range) [The point is that $4n = 1$ has no solution in $\mathbb{Z}$]

(4) $f: \mathbb{R} \to \mathbb{R}$, $x \to 4x$, $\frac{y}{4} \to y$

a) f is one to one. Let $f(a) = f(b)$ where a and b are real numbers. Then $4a = 4b$, so $a = b$.

b) f is onto. Let $y \in \mathbb{R}$, then $\frac{y}{4} \in \mathbb{R}$ and $f(\frac{y}{4}) = 4 \cdot \frac{y}{4} = y$

[The main point is that $y = 4x$ has solution in $\mathbb{R}$ $x = \frac{y}{4}$]

(5) $f: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$, $f(m,n) = m^2 + n^2$

a) f is not one to one. $f(1,0) = 1$ $f(0,1) = 1$ $(1,0) \neq (0,1)$. So 2 different inputs have the same output

b) f is not onto. The range is the set of integers that can be written as sum of squares, which is different from the codomain $\mathbb{Z}$.

(or: $3 \in$ codomain, but $3 \notin$ range since $m^2 + n^2 = 3$ has no solution in $\mathbb{Z} \times \mathbb{Z}$)

(6) $a_n = 2^n + (-2)^n$

$a_1 = 2 + (-2) = \boxed{0}$

$a_2 = 4 + (-2)^2 = \boxed{8}$

$a_3 = 2^3 + (-2)^3 = 8 - 8 = \boxed{0}$

$a_4 = 2^4 + (-2)^4 = 16 + 16 = \boxed{32}$

(7) a) $\left\lceil -\frac{1}{3} \right\rceil = 0$

b) $\left\lfloor -\frac{1}{3} \right\rfloor = -1$

c) $\sum_{n=2}^{4} (-3)^n = (-3)^2 + (-3)^3 + (-3)^4 = 9 - 27 + 81 = \boxed{63}$

d) $\sum_{i=0}^{2} \sum_{j=0}^{3} (2i + 3j) = \sum_{i=0}^{2} (2i + 2i + 3 + 2i + 6 + 2i + 9) =$

$= \sum_{i=0}^{2} (8i + 18) = 18 + 8 + 18 + 16 + 18 = \boxed{78}$

(8) x is congruent to 4 modulo 11 if $x-4$ is a multiple of 11

a) Yes, $59-4=55$ and 55 is a multiple of 11

b) No, $51-4=47$ which is not a multiple of 11

c) No, $-59-4=-63$ which is not a multiple of 11

d) Yes, $-51-4=-55$ which is a multiple of 11

e) Yes, $4-4=0$ which is a multiple of 11 $(0=0\cdot 11)$

(9) a) $144 \bmod 7 = \boxed{4}$ $\qquad 144 = 7(20)+4$

b) $(-94 \bmod 5) \bmod 3 = \boxed{1}$ $\qquad -94 = 5(-19)+1$

$-94 \bmod 3 = 1$

$1 \bmod 3 = 1$

(10) Let n be a multiple of 3. Then $n=3k$, where k is an integer. Then $n^3+45 = (3k)^3+45 = 27k^3+45 = 9(3k^3+5)$ where $3k^2+5$ is an integer. So n^3+45 is a multiple of 9