Mathematics 1275/D500, Spring 2017 **Exam #4**
Instructor: Suman Ganguli **Wednesday, May 17** **Name:** ____________________

Question:	1	2	3	4	5	Total
Points:	10	10	10	10	10	50
Score:						

In order to receive full credit, you must **show all your work** and simplify your answers.

1. (10 points) Solve the system of equations

$$7x + y^2 = 1 \quad (1)$$
$$x^2 - y^2 = -11 \quad (2)$$

according to the following steps:

(a) Use the addition method to eliminate the variable y, which will reduce the system to a single quadratic equation involving only x.

Solution: Adding equations times (1) and (2) yields $x^2+7x = -10$, i.e., the quadratic equation $x^2+7x+10 = 0$.

(b) Solve the quadratic equation from part (a). (You can solve by factoring or by using the quadratic formula.)

Solution: Since $x^2 + 7x + 10 = (x + 5)(x + 2)$, the two solutions of $x^2 + 7x + 10 = 0$ are $x = -5$ and $x = -2$. The quadratic formula yields the same solutions:

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(10)}}{2(1)} = \frac{-7 \pm \sqrt{9}}{2} = \frac{-7 \pm 3}{2}$$

i.e., $x = \frac{-7+3}{2} = \frac{-4}{2} = -2$ and $x = \frac{-7-3}{2} = \frac{-10}{2} = -5$.

(c) For each of the x-values from part (b), solve for the corresponding value(s) of y. (You can use either of the original equations (1) or (2).)

Solution: From (1), we have $y^2 = 1 - 7x$, so

$$x = -5 \Longrightarrow y^2 = 1 - 7(-5) = 1 + 35 = 36 \Longrightarrow y = \pm\sqrt{36} = \pm 6$$

and

$$x = -2 \Longrightarrow y = 1 - 7(-2) = 1 + 14 = 15 \Longrightarrow y = \pm\sqrt{15}$$

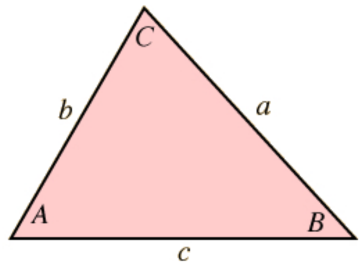
(d) Write the solutions as points (x, y).

Solution: The solutions to the system of equations are $(-5, 6), (-5, -6), (-2, \sqrt{15})$ and $(-2, -\sqrt{15})$.

2. (10 points) For each triangle $\triangle ABC$, solve using the given information and either the Law of Sines or the Law of Cosines. Round each answer to the nearest tenth (i.e., to one decimal place).

Law of Sines: For any triangle (i.e., not necessarily a right triangle) with sidelengths a, b, c and corresponding angles A, B, C (as shown)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines: For any such triangle,

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ b^2 &= a^2 + c^2 - 2ac\cos B \\ c^2 &= a^2 + b^2 - 2ab\cos C \end{aligned}$$

(a) Suppose $b = 7$ in, $c = 9$ in, and $\angle A = 52°$. Solve for the length of side a.

Solution: By the Law of Cosines,

$$a^2 = b^2 + c^2 - 2bc\cos A = 7^2 + 9^2 - 2(7)(9)\cos 52° = 49 + 81 - 126\cos 52° \approx 130 - 126(0.61566) \approx 52.4$$

Hence, $c \approx \sqrt{52.4} \approx 7.2$ in.

(b) Suppose $\angle A = 33°$, $\angle C = 102°$, and $c = 12$ cm. Solve for the length of side a.

Solution: By the Law of Sines,

$$\frac{a}{\sin 33°} = \frac{12}{\sin 102°} \Longrightarrow a = \frac{12\sin 33°}{\sin 102°} \approx \frac{12(0.5446)}{0.9781} \approx \frac{6.54}{0.9781} \approx 6.7 \text{ cm}$$

3. (10 points) Solve the following system of 3 linear equations in 3 variables:

$$\begin{aligned} -3x + y + z &= 10 \quad (1) \\ -x - 2y - 3z &= 10 \quad (2) \\ -2x + 2y + z &= 6 \quad (3) \end{aligned}$$

(a) Choose a pair of equations, and eliminate one of the variables by using the addition method. (Hint: You can eliminate y by adding equations (2) and (3)!)

Solution: Add equations (2) and (3) in order to eliminate x:

$$\begin{array}{r} -x - 2y - 3z = 10 \\ -2x + 2y + z = 6 \\ \hline -3x - 2z = 16 \end{array}$$

(b) Choose a *different* pair of the original equations, and eliminate the *same* variable:

Solution: Add 2*(1) to (2) in order to eliminate y:

$$\begin{array}{r} -6x + 2y + 2z = 20 \\ -x - 2y - 3z = 10 \\ \hline -7x - z = 30 \end{array}$$

(c) Solve the resulting system of 2 equations in 2 variables:

Solution: First, multiply through the equation $-7x - z = 30$ from step (b) by -2 and add that to the equation $-3x - 2z = 16$ from step (a), in order to eliminate z:

$$\begin{array}{r} -3x - 2z = 16 \\ 14x + 2z = -60 \\ \hline 11x = -44 \end{array}$$

That yields $x = -4$. Substituting this into either equation above lets us solve for z, e.g., substituting $x = -4$ into $14x + 2z = -60$:

$$14(-4) + 2z = -60 \Longrightarrow -56 + 2z = -60 \Longrightarrow 2z = -60 + 56 = -4 \Longrightarrow z = -2$$

(d) Substitute the values of the 2 variables you found in part (c) into any of the 3 original equations, and solve for the 3rd variable.

Solution: Substituting $x = -4$ and $z = -2$ into equation (1):

$$-3(-4) + y - 2 = 10 \Longrightarrow 12 + y - 2 = 10 \Longrightarrow y = 0$$

(e) Write down the solution to the system:

$x =$

$y =$

$z =$

4. (10 points) Evaluate the following logarithms. (Recall that $\log_b y = x$ if $b^x = y$.)

(a)

$$\log_9 81 =$$

Solution: Since $9^2 = 81$,

$$\log_9 81 = 2$$

(b)

$$\log_{10} 1,000,000 =$$

Solution: Since $1,000,000 = 10^6$,

$$\log_{10} 1,000,000 = 6$$

(c)

$$\log_3 \sqrt{3} =$$

Solution: Since $\sqrt{3} = 3^{1/2}$

$$\log_3 \sqrt{3} = \frac{1}{2}$$

(d)

$$\log_2 \frac{1}{8} =$$

Solution: Since $\frac{1}{8} = \frac{1}{2^3} = 2^{-3}$

$$\log_2 \frac{1}{8} = -3$$

(e)

$$\log_5 0.2 =$$

Solution: Since $0.2 = \frac{1}{5} = 5^{-1}$

$$\log_5 0.2 = -1$$

5. (10 points) Prove each of the following trig identities below, using the fundamental trig identities:

$$\sin^2\theta + \cos^2\theta = 1 \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\cot^2\theta + 1 = \csc^2\theta \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}\,,\ \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

(a)

$$\tan\theta + \cot\theta = \sec\theta\csc\theta$$

(Hint: Start by rewriting everything in terms of $\sin\theta$ and $\cos\theta$.)

Solution: Rewriting everything in terms of $\sin\theta$ and $\cos\theta$ yields:

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \stackrel{?}{=} \frac{1}{\sin\theta}\frac{1}{\cos\theta}$$

Then we add the ratios on the LHS using a LCD of $\sin\theta\cos\theta$:

$$\begin{aligned}\frac{\sin\theta}{\cos\theta}\frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\frac{\cos\theta}{\cos\theta} &\stackrel{?}{=} \frac{1}{\sin\theta}\frac{1}{\cos\theta}\\ \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta} &\stackrel{?}{=} \frac{1}{\sin\theta}\frac{1}{\cos\theta}\\ \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta} &\stackrel{?}{=} \frac{1}{\sin\theta}\frac{1}{\cos\theta}\end{aligned}$$

Now we can apply the fundamental identity $\sin^2\theta + \cos^2\theta = 1$ to the numerator of the LHS, showing that the two sides are in fact equal:

$$\frac{1}{\sin\theta\cos\theta} \stackrel{\checkmark}{=} \frac{1}{\sin\theta}\frac{1}{\cos\theta}$$

(b)

$$\tan\theta\csc\theta\sec\theta = 1 + \tan^2\theta$$

(Hint: start by rewriting the LHS in terms of $\sin\theta$ and $\cos\theta$. You can do that on the RHS as well, or you can apply one of the fundamental identities to the RHS.)

Solution: Rewriting the LHS in in terms of $\sin\theta$ and $\cos\theta$ and applying the identity $\tan^2\theta + 1 = \sec^2\theta$ to the RHS yields:

$$\frac{\sin\theta}{\cos\theta}\frac{1}{\sin\theta}\frac{1}{\cos\theta} \stackrel{?}{=} \sec^2\theta$$

Simplifying the LHS by cancelling the common factor $\sin\theta$ and noting that $\sec^2\theta = \dfrac{1}{\cos^2\theta}$ for the RHS establishes that the two sides of the identity are in fact equal:

$$\frac{1}{\cos\theta}\frac{1}{\cos\theta} \stackrel{\checkmark}{=} \frac{1}{\cos^2\theta}$$