

Solve the system of equations according to the following steps:

$$x^2 - y = 3 \quad (1)$$

$$2x + y = 5 \quad (2)$$

1. Use the addition method to reduce the system to a single equation involving only x .

(Hint: You can eliminate y by adding equations (1) and (2), resulting in a quadratic equation in x .)

Solution: Adding equations (1) and (2) yields $x^2 + 2x = 8$, i.e., the quadratic equation $x^2 + 2x - 8 = 0$.

2. Show that you get the same quadratic equation in x by using the substitution method (i.e., use either equation to solve for y in terms of x , and then substitute for y into the other equation).

Solution: Solving for y from equation (2) yields $y = 5 - 2x$. So we substitute $5 - 2x$ for y in equation (1):

$$x^2 - (5 - 2x) = 3 \implies x^2 + 2x - 8 = 0$$

3. Solve the quadratic equation from parts (a) and (b) for x . (You can solve by factoring or by using the quadratic equation. You should get two integer solutions for x .)

Solution: Since $x^2 + 2x - 8 = (x + 4)(x - 2)$, the two solutions of $x^2 + 2x - 8 = 0$ are $x = -4$ and $x = 2$.

4. For each of the x -values, solve for the corresponding values of y . (Use either of the original equations (1) or (2); in particular, in #2 you should have solved for y in terms of x —use that!)

Solution: Using $y = 5 - 2x$ from #2 (which was derived from equation (2)):

$$x = -4 \implies y = 5 - 2(-4) = 13$$

$$x = 2 \implies y = 5 - 2(2) = 1$$

So the solutions to the system of equations are $(-4, 13)$ and $(2, 1)$.