Mathematics 1275/D500, Spring 2017 **Exam #3**
Instructor: Suman Ganguli **Wednesday, May 3** **Name:** ____________________

Question:	1	2	3	4	5	Total
Points:	5	10	15	10	10	50
Score:						

In order to receive full credit, you must **show all your work** and simplify your answers.

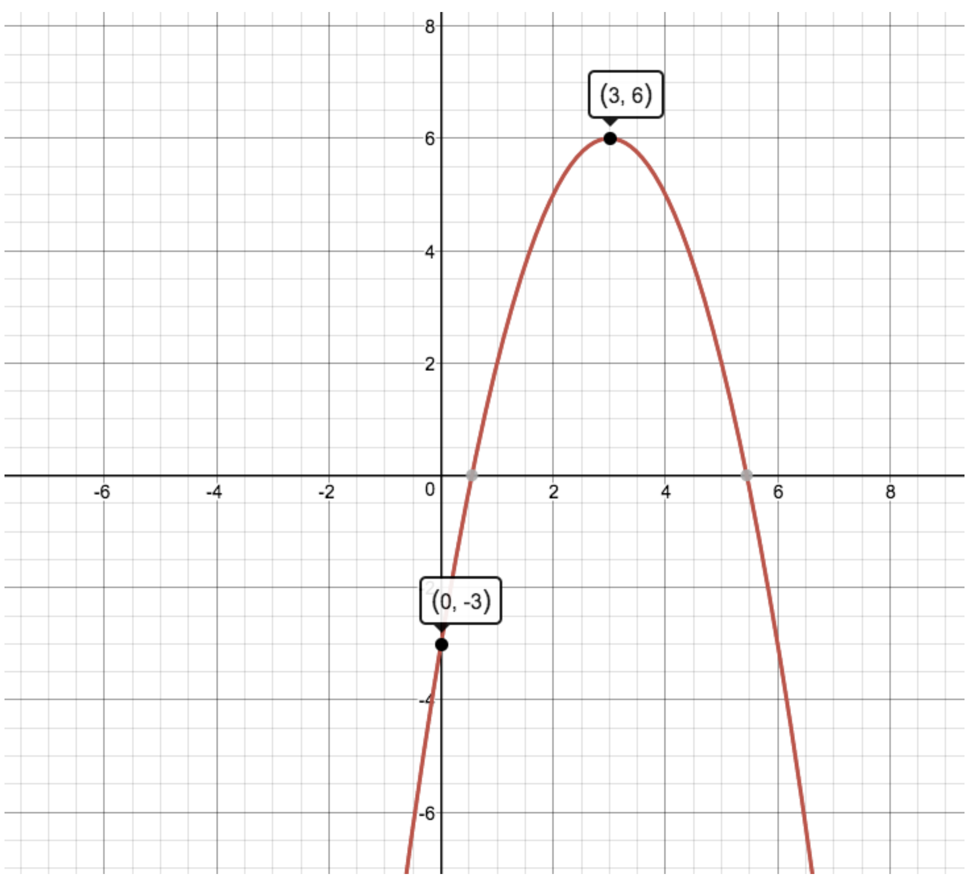
1. (5 points) Identify the vertex and y-intercept, and then sketch the graph.
(Hint: in order to find the vertex, complete the square. In order to find the y-intercept, plug in $x = 0$.)

$$y = -x^2 + 6x - 3$$

Solution: In order to find the vertex, we complete the square:

$$y = -(x^2 - 6x) - 3 = -(x^2 - 6x + 9) - 3 + 9 = -(x - 3)^2 + 6$$

Hence, the vertex is $(3, 6)$. Plugging in $x = 0$ yields $y = -0^2 + 6(0) - 3 = 0 + 0 - 3 = -3$, so the y-intercept is $(0, -3)$. Because the leading term is negative, the parabola opens downwards.

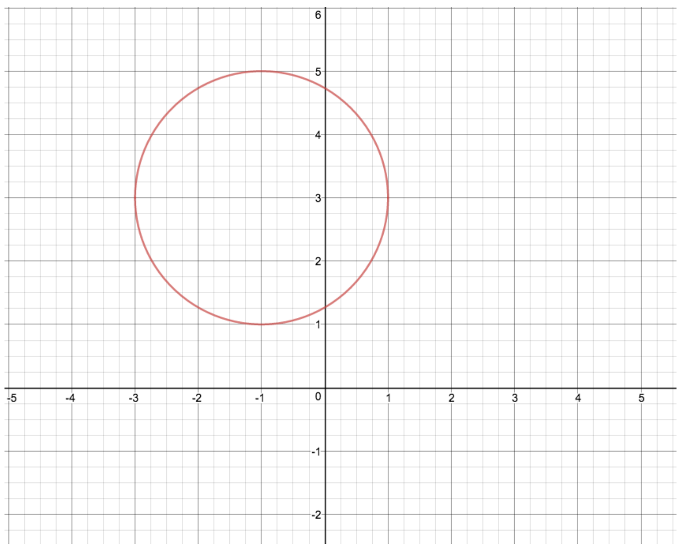


2. (10 points) Identify the center and radius for each circle. Use the center and radius to identify four points on the circle, and use these to sketch the graph of the circle. **Label the center and the four points on the circle on your graph.** (Recall that the standard form of the equation of a circle centered at (h, k) with radius r is $(x - h)^2 + (y - k)^2 = r^2$.)

(a)

$$(x + 1)^2 + (y - 3)^2 = 4$$

Solution: The center of the circle is $(-1, 3)$, with radius $r = \sqrt{4} = 2$. Hence, four points on the circle are: $(-1, 5), (-1, 1), (-3, 3), (1, 3)$.



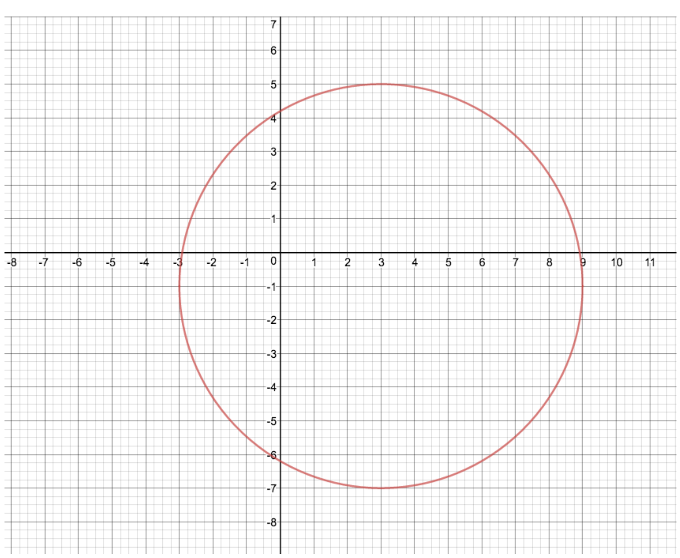
(b)

$$x^2 + y^2 - 6x + 2y - 26 = 0$$

Solution: Completing the square on the x-terms and y-terms separately:

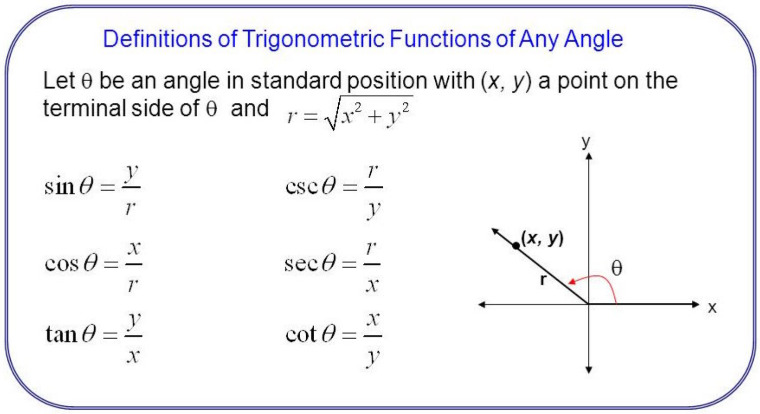
$$(x^2 - 6x + 9) + (y^2 + 2y + 1) = 26 + 9 + 1 \Longrightarrow (x - 3)^2 + (y + 1)^2 = 36$$

Hence, the circle is centered at $(3, -1)$, with radius $r = \sqrt{36} = 6$, and so four points on the circle are: $(3, 5), (3, -7), (9, -1), (-3, -1)$.



3. (15 points) Use the given information to find the values of the trigonometric functions for the angle θ:

- first use the signs of the given trig functions to determine which quadrant the angle θ is in;
- then use the given trig value to identify values of x, y, and r;
- finally, use the following definitions to find the values of the five other trig functions:



(a) $\cos\theta = \dfrac{1}{2}$ and $\tan\theta > 0$:

Solution: Since both $\cos\theta > 0$ and $\tan\theta > 0$, θ must be in Quadrant I. Since $\cos\theta = \dfrac{x}{r} = \dfrac{1}{2}$, $x = 1$ and $r = 2$. We solve for y using the Pythagorean Theorem $r^2 = x^2 + y^2$, which means $y = \sqrt{r^2 - x^2}\sqrt{2^2 - 1^2} = \sqrt{3}$.
So the values of the trig functions are:

$$\sin\theta = \frac{\sqrt{3}}{2} \qquad \csc\theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cos\theta = \frac{1}{2} \qquad \sec\theta = 2$$

$$\tan\theta = \sqrt{3} \qquad \cot\theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

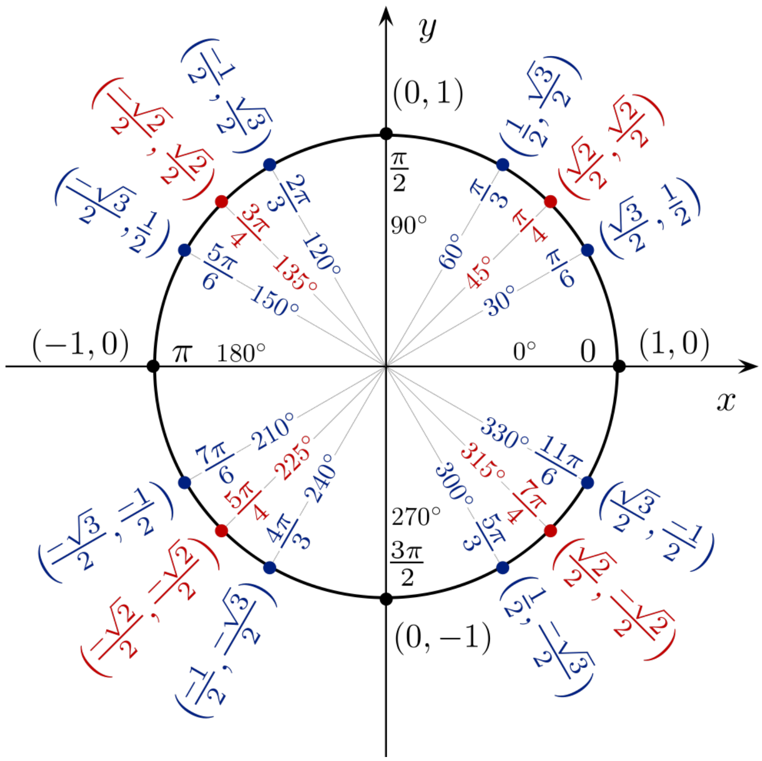
(b) $\tan\theta = -3$ and $\cos\theta > 0$:

Solution: Since $\tan\theta < 0$ and $\cos\theta > 0$, θ must be in Quadrant IV. Note that since θ is in Quadrant IV, $x > 0$ and $y < 0$. Therefore, $\tan\theta = \dfrac{y}{x} = -3$ implies $y = -3$ and $x = 1$. Hence, $r = \sqrt{x^2 + y^2} = \sqrt{1^2 + (-3)^2} = \sqrt{10}$.
So the values of the trig functions are:

$$\sin\theta = \frac{-3}{\sqrt{10}} = -\frac{3\sqrt{10}}{10} \qquad \csc\theta = -\frac{\sqrt{10}}{3}$$

$$\cos\theta = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} \qquad \sec\theta = \sqrt{10}$$

$$\tan\theta = -3 \qquad \cot\theta = -\frac{1}{3}$$



4. (10 points) Use the unit circle to find each of the following (express each answer in exact form, not as a decimal). (Recall that for a point (x, y) on the unit circle corresponding to an angle θ, $x = \cos\theta$ and $y = \sin\theta$.)

(a) **Solution:** $\cos 150° = -\dfrac{\sqrt{3}}{2}$

(b) **Solution:** $\sin\dfrac{3\pi}{4} = \dfrac{\sqrt{2}}{2}$

(c) **Solution:** $\tan 300° = \dfrac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$

(d) **Solution:** $\sec\dfrac{\pi}{6} = \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

(e) **Solution:** $\cot\dfrac{4\pi}{3} = \dfrac{-1/2}{-\sqrt{3}/2} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

5. (10 points) Find all the solutions θ of the following equations, for $0 \le \theta < 2\pi$. (Use the unit circle on the previous page!)

(a) $4\sin\theta = 8$

Solution: $4\sin\theta = 8 \iff \sin\theta = \frac{1}{2}$, so we look for the angles θ for which the y-coordinate of the point on the unit circle is $\frac{1}{2}$. Hence, the solutions are $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

(b) $-6\cos\theta = 3\sqrt{2}$

Solution: $-6\cos\theta = 3\sqrt{2} \iff \cos\theta = -\frac{\sqrt{2}}{2}$ so we look for the angles θ for which the x-coordinate of the point on the unit circle is $-\frac{\sqrt{2}}{2}$. Hence, the solutions are $\theta = \frac{3\pi}{4}$ and $\theta = \frac{5\pi}{4}$.