Mathematics 1275/D500, Spring 2017 **Exam #2**
Instructor: Suman Ganguli **Monday, April 3** **Name:** ____________________

Question	Points	Score
1	5	
2	15	
3	15	
4	5	
5	10	
Total:	50	

In order to receive full credit, you must **show all your work** and simplify your answers.

1. (5 points) Simplify the following complex fraction:

$$\frac{\frac{2}{x^2} - \frac{6}{x}}{4 + \frac{2}{x^2}} =$$

Solution: Method I:

$$\frac{\frac{2}{x^2} - \frac{6x}{x^2}}{\frac{4x^2}{x^2} + \frac{2}{x^2}} = \frac{\frac{2-6x}{x^2}}{\frac{4x^2+2}{x^2}} = \frac{2-6x}{x^2} \cdot \frac{x^2}{4x^2+2} = \frac{2-6x}{4x^2+2} = \frac{1-3x}{2x^2+1}$$

Method II, using the LCD of all individual terms x^2:

$$\frac{\frac{2}{x^2} - \frac{6}{x}}{4 + \frac{2}{x^2}} \cdot \frac{x^2}{x^2} = \frac{\frac{2x^2}{x^2} - \frac{6x^2}{x}}{4x^2 + \frac{2x^2}{x^2}} = \frac{2-6x}{4x^2+2} = \frac{1-3x}{2x^2+1}$$

2. (15 points) Perform the indicated operations on the complex numbers. Write the result in standard complex form, i.e., in the form $a + bi$.

(a)

$$(-2-3i)-(-7+5i)$$

Solution:

$$(-2-3i)-(-7-5i)=(-2+7)+(-3i-5i)=5-8i$$

(b)

$$(4+2i)(1+i)$$

Solution:

$$(4+2i)(1+i)=4+4i+2i+2i^2=4+6i-2=2+6i$$

(c)

$$(-2-3i)(-7-5i)$$

Solution:

$$(-2-3i)(-7-5i)=14+10i+21i+15i^2=14+31i-15=-1+31i$$

(d)

$$\frac{5}{3+i}$$

Solution:

$$\frac{5}{3+i}\times\frac{3-i}{3-i}=\frac{15-5i}{9+3i-3i-i^2}=\frac{15-5i}{9-(-1)}=\frac{15}{10}-\frac{5i}{10}=\frac{3}{2}-\frac{1}{2}i$$

(e)

$$\frac{1+8i}{1-2i}$$

Solution:

$$\frac{1+8i}{1-2i}\times\frac{1+2i}{1+2i}=\frac{1+2i+8i+(8i)(2i)}{1+2i-2i-4i^2}=\frac{1+10i-16}{1+4}=\frac{-15+10i}{5}=\frac{-15}{5}+\frac{10i}{5}=-3+2i$$

Check: $(-3+2i)(1-2i)=-3-3(-2i)+2i-4i^2=-3+6i+2i+4=1+8i$✓

3. (15 points) Use the quadratic formula to solve for x. Simplify the solutions completely.

(a)

$$2x^2 + 8x + 10 = 0$$

Solution:
Applying the quadratic formula with $a = 2, b = 8, c = 10$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{8^2 - 4(2)(10)}}{2(2)} = \frac{-8 \pm \sqrt{64 - 80}}{4} = \frac{-8 \pm \sqrt{-16}}{4} = -\frac{8}{4} \pm \frac{4i}{4} = -2 \pm i$$

(b)

$$x^2 + 6x + 2 = 0$$

Solution:
Applying the quadratic formula with $a = 1, b = 6, c = 2$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4(1)(2)}}{2(1)} = \frac{-6 \pm \sqrt{28}}{2} = \frac{-6 \pm 2\sqrt{7}}{2} = -3 \pm \sqrt{7}$$

(c)

$$3x^2 - 5x + 2 = 0$$

Solution:
Applying the quadratic formula with $a = 3, b = -5, c = 2$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(2)}}{2(3)} = \frac{5 \pm \sqrt{25 - 24}}{6} = \frac{5 \pm 1}{6}$$

so the solutions are $x = \frac{6}{6} = 1$ and $x = \frac{4}{6} = \frac{2}{3}$.

4. (5 points) Solve the following equation (the same equation as in #2(b)!)

$$x^2 + 6x + 2 = 0$$

by completing the square and using the square root property:

(a) First, separate the constant term from the variable terms (i.e., move the constant term from the LHS to the RHS):

Solution: $x^2 + 6x = -2$

(b) Now complete the square on the LHS, i.e., compute the number that will make the LHS into a perfect square trinomial, and add it to both sides of the equation:

$$x^2 + 6x + \_\_\_ = \_\_\_ + \_\_\_$$

Solution: $x^2 + 6x + 9 = -2 + 9$

(c) Simplify the equation by factoring the perfect square trinomial on the LHS, and adding the numbers on the RHS:

Solution: $(x + 3)^2 = 7$

(d) Now take the square root of both sides, remembering the square root property (i.e., there are two square roots to consider on the RHS, the positive and the negative!):

Solution: $x + 3 = \pm\sqrt{7}$

(e) Finally, solve for x:

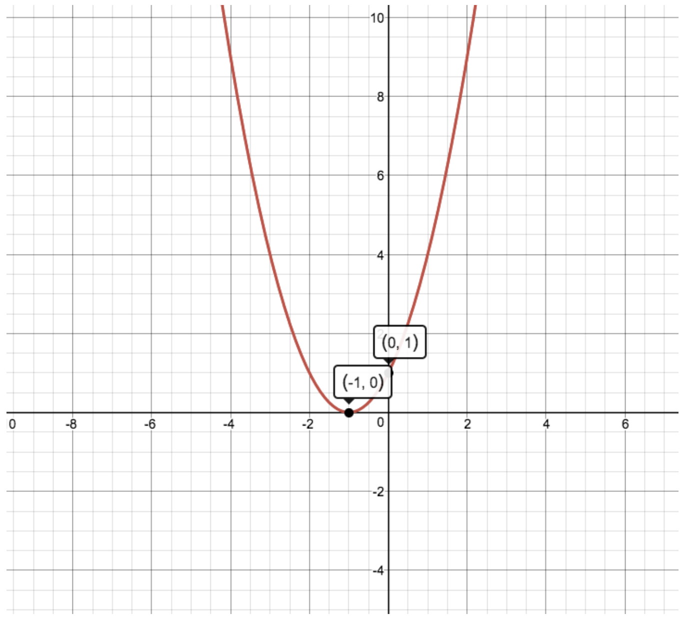
Solution: $x = -3 \pm \sqrt{7}$

5. (10 points) Identify the vertex and y-intercept for each parabola, and then sketch the graph.

(a)

$$y = (x+1)^2$$

Solution: Since $y = (x+1)^2$, the graph consists of $y = x^2$ shifted to the left one unit, and hence the vertex is $(-1, 0)$. Also note that the y-intercept is $(0, 1)$ (since $y = 0^2 + 2(0) + 1 = 1$ when $x = 0$).



(b)

$$y = x^2 + 6x + 2$$

Solution: We find the vertex by completing the square on $x^2 + 6x$:

$$y = (x^2 + 6x + 9) + 2 - 9 = (x+3)^2 - 7$$

so the vertex is $(-3, -7)$. The y-intercept is $(0, 2)$. Also useful for graphing are the x-intercepts, which from #2(b) are at $x = -3 + \sqrt{7} \approx -3 + 2.65 = -0.35$ and $x = -3 - \sqrt{7} \approx -3 - 2.65 = -5.65$.