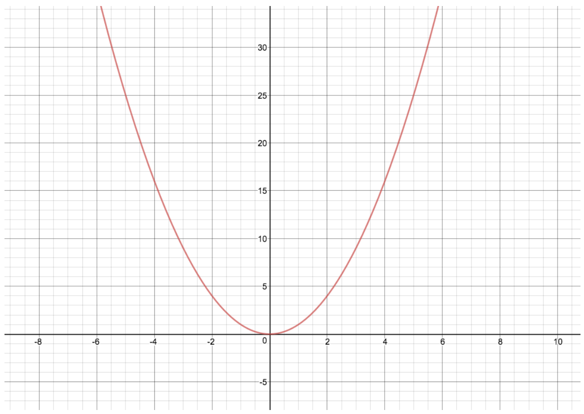
Sec 7.4: Graphing Quadratic Functions

Basic graph: the parabola $y = x^2$, with vertex at the the origin $(0, 0)$:

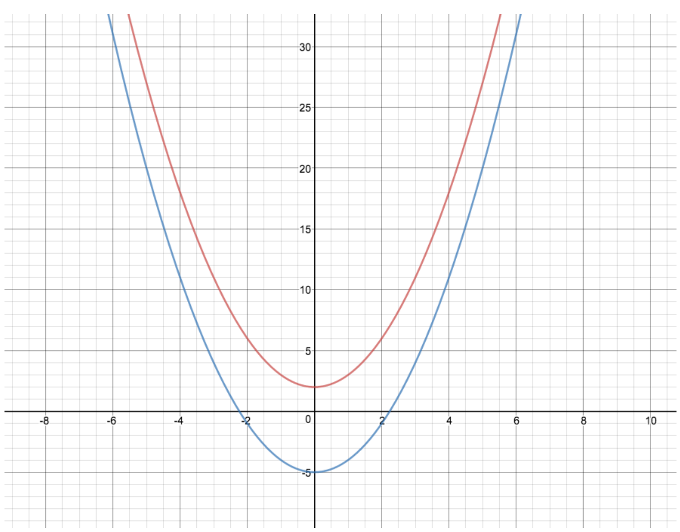


Vertical shifts: $y = x^2 \pm k$

If a constant k is added or subtracted to/from x^2, that shifts the graph of $y = x^2$ vertically:

- The graph of $y = x^2 + k$ is just the graph of $y = x^2$ shifted **up** k units.
- The graph of $y = x^2 - k$ is just the graph of $y = x^2$ shifted **down** k units.
  - Read Sec 7.4, pp605-606 (Examples 1 and 2)

Example 1: Shown are the graphs of $y = x^2 + 2$ and $y = x^2 - 5$. Label the vertex, the y-intercept, and the x-intercepts (if any) on each graph.



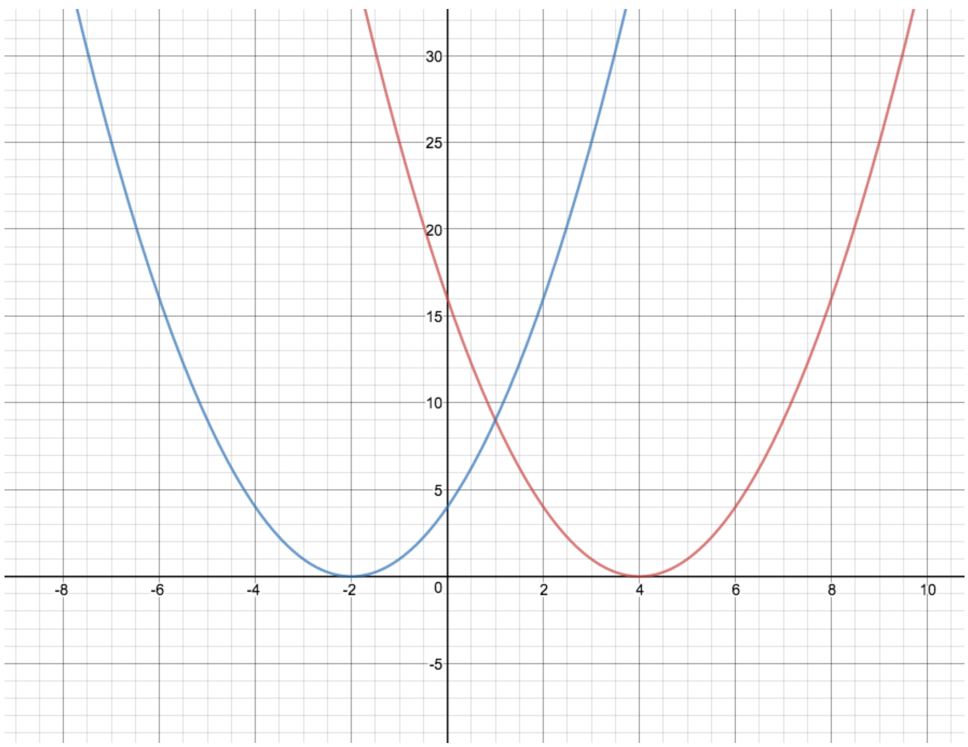
- To find the y-intercept of a graph, plug in $x = 0$.
- To find the x-intercept(s), solve the equation $y = 0$ for x.

Horizontal shifts: $y = (x \pm h)^2$

If a constant h is added or subtracted to/from x and then squared, that shifts the graph of $y = x^2$ horizontally:

- The graph of $y = (x - h)^2$ is the graph of $y = x^2$ shifted **right** h units.
- The graph of $y = (x + h)^2$ is just the graph of $y = x^2$ shifted **left** h units.
  - Read Sec 7.4, pp607-608 (Examples 3 and 4)

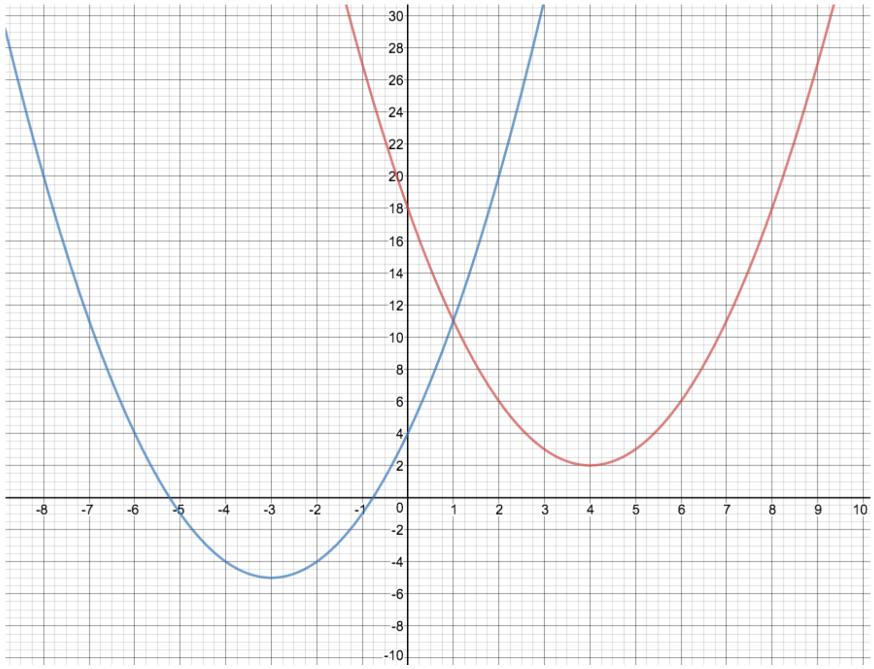
Example 2: Shown are the graphs of $y = (x - 4)^2$ and $y = (x + 2)^2$. Label the vertex, the y-intercept, and the x-intercepts (if any) on each graph.



Vertical *and* horizontal shifts: $y = (x \pm h)^2 \pm k$

- The graph of a function of the form $y = (x \pm h)^2 \pm k$ is a parabola which is
  - shifted vertically k units (up or down depending on the sign in front of the vertical shift k), *and*
  - shifted horizontally h units (left or right depending on the sign in front of the horizontal shift k)

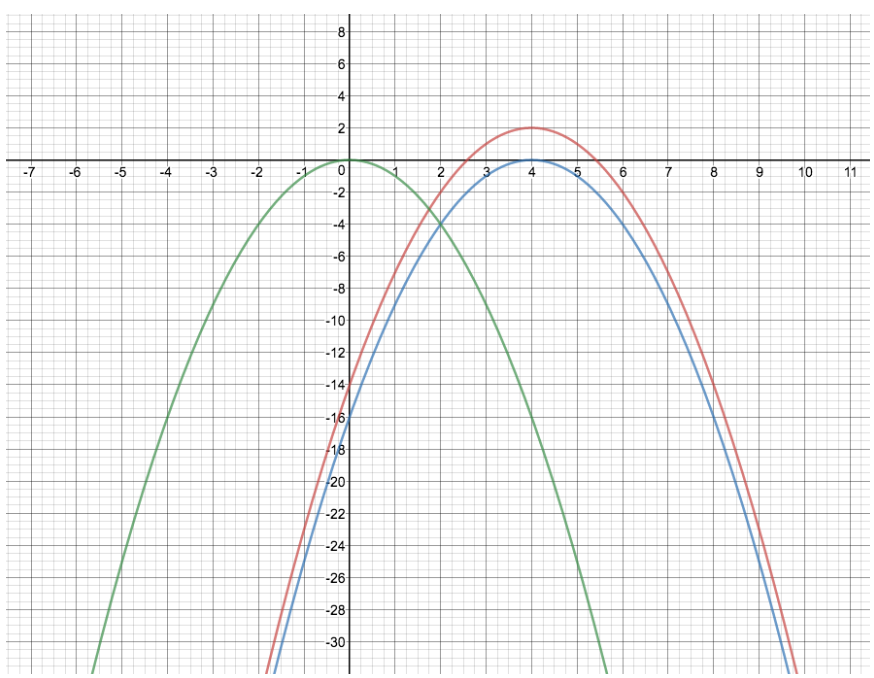
Example 3: Shown are the graphs of $y = (x-4)^2 + 2$ and $y = (x+3)^2 - 5$. Label the vertex and the y-intercept on each graph. How could you find the x-intercepts?



"Upside-down" parabolas: $y = -(x \pm h)^2 \pm k$

- The negative sign flips the basic parabola shape upside-down, i.e., so that it opens downward
- Read Sec 7.4, Example 8

Example 4: Shown are the graphs of $y = -x^2$, $y = -(x-4)^2$ and $y = -(x-4)^2 + 2$.



Sec 7.5: Writing a Quadratic in the form $y = (x \pm h) \pm k$

Now given any quadratic $y = x^2 + bx + c$, we can write it in the form $y = (x \pm h)^2 \pm k$ by completing the square!

Example 5: Suppose we want to sketch the graph of $y = x^2 + 8x + 13$.

1. Write $y = x^2 + 8x + 13$ in the form $y = (x \pm h)^2 \pm k$ by completing the square:

$$y = x^2 + 8x + 13 = (x^2 + 8x + \qquad) + 13 \qquad =$$

2. Identify the horizontal and vertical shifts of the graph, and hence the vertex of the parabola:

   - horizontal shift:
   - vertical shift:
   - vertex:

3. Identify the y-intercept of the graph:

4. Identify the x-intercepts of the graph by using the quadratic formula:

5. Use this information to sketch the graph of $y = x^2 + 8x + 13$: