**Mathematics 1275/D500, Spring 2017** **Exam #1**
**Instructor: Suman Ganguli** **Wednesday, March 1** **Name:** ____________________

| Question | Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 15 | |
| 5 | 5 | |
| Total: | 50 | |

In order to receive full credit, you must **show all your work** and simplify your answers.

1. (10 points) Simplify the following expressions, writing the results using only positive integer exponents:

(a)

$$\left(2x^{-5}y^{2}\right)^{-3} =$$

**Solution:**

$$\left(2x^{-5}y^{2}\right)^{-3} = \frac{1}{\left(2x^{-5}y^{2}\right)^{3}} = \frac{1}{8x^{-15}y^{6}} = \frac{x^{15}}{8y^{6}}$$

(b)

$$\left(\frac{6xy^{-3}}{4x^{2}y}\right)^{2} =$$

**Solution:**

$$\left(\frac{6xy^{-3}}{4x^{2}y}\right)^{2} = \frac{36x^{2}y^{-6}}{16x^{4}y^{2}} = \frac{9}{4x^{2}y^{8}}$$

or

$$\left(\frac{6xy^{-3}}{4x^{2}y}\right)^{2} = \left(\frac{3}{2xy^{4}}\right)^{2} = \frac{9}{4x^{2}y^{8}}$$

2. (10 points) Add or subtract the rational expressions. (Hint: start by factoring the denominators in order to identify the LCD.)

(a)

$$\frac{3y}{y^2-49} - \frac{1}{7-y} =$$

**Solution:** Since $y^2 - 49 = (y-7)(y+7)$ and $7 - y = -(y-7)$, the LCD is $(y-7)(y+7)$:

$$\frac{3y}{y^2-49} - \frac{1}{7-y} = \frac{3y}{(y-7)(y+7)} - \frac{-1}{y-7} = \frac{3y}{(y-7)(y+7)} + \frac{1}{y-7}$$

$$= \frac{3y}{(y-7)(y+7)} + \frac{1}{(y-7)} \cdot \frac{(y+7)}{(y+7)} = \frac{3y+(y+7)}{(y+7)(y-7)} = \frac{4y+7}{(y+7)(y-7)}$$

(b)

$$\frac{2}{3x-15} + \frac{x}{x^2-6x+5} =$$

**Solution:** Since $3x - 15 = 3(x-5)$ and $x^2 - 6x + 5 = (x-5)(x-1)$, the LCD is $3(x-5)(x-1)$:

$$\frac{2}{3(x-5)} + \frac{x}{(x-5)(x-1)} = \frac{2(x-1)}{3(x-5)(x-1)} + \frac{3x}{3(x-5)(x-1)} =$$

$$\frac{2x-2}{3(x-5)(x-1)} + \frac{3x}{3(x-5)(x-1)} = \frac{5x-2}{3(x-5)(x-1)}$$

3. (10 points) Simplify the following complex fractions:

(a)

$$\frac{\frac{1}{9x}-\frac{1}{3}}{1-\frac{1}{3x}} =$$

**Solution:** Method I:

$$\frac{\frac{1}{9x}-\frac{3x}{9x}}{\frac{3x}{3x}-\frac{1}{3x}} = \frac{\frac{1-3x}{9x}}{\frac{3x-1}{3x}} = \frac{1-3x}{9x}\cdot\frac{3x}{3x-1} = \frac{1}{3}\cdot\frac{1-3x}{3x-1} = -\frac{1}{3}$$

Method II, using the LCD of all individual terms $\frac{1}{9x}$:

$$\frac{\frac{1}{9x}-\frac{1}{3}}{1-\frac{1}{3x}}\cdot\frac{9x}{9x} = \frac{\frac{9x}{9x}-\frac{9x}{3}}{9x-\frac{9x}{3x}} = \frac{1-3x}{9x-3} = \frac{1-3x}{3(3x-1)} = -\frac{1}{3}$$

(b)

$$\frac{x-x^{-1}}{1-x^{-2}} =$$

**Solution:**

First, note that $\frac{x-x^{-1}}{1-x^{-2}} = \frac{x-\frac{1}{x}}{1-\frac{1}{x^2}}$

Method I:

$$\frac{x-\frac{1}{x}}{1-\frac{1}{x^2}} = \frac{\frac{x^2}{x}-\frac{1}{x}}{\frac{x^2}{x^2}-\frac{1}{x^2}} = \frac{\frac{x^2-1}{x}}{\frac{x^2-1}{x^2}} = \frac{x^2-1}{x}\cdot\frac{x^2}{x^2-1} = \frac{x^2}{x} = x$$

Method II: use the LCD of all individual terms, which is $x^2$:

$$\frac{x-\frac{1}{x}}{1-\frac{1}{x^2}}\cdot\frac{x^2}{x^2} = \frac{x^3-\frac{x^2}{x}}{x^2-\frac{x^2}{x^2}} = \frac{x^3-x}{x^2-1} = \frac{x(x^2-1)}{x^2-1} = x$$

4. (15 points) Simplify the following radical expressions:

(a)

$$\sqrt{48x^8y^3} =$$

**Solution:**

$$\sqrt{48x^8y^3} = \sqrt{16x^8y^2 \cdot 3y} = 4x^4y\sqrt{3y}$$

(b)

$$5\sqrt{32} + 2\sqrt{50} =$$

**Solution:**

$$5\sqrt{32} + 2\sqrt{50} = 5\sqrt{16 \cdot 2} + 2\sqrt{25 \cdot 2} = (5 \cdot 4)\sqrt{2} + (2 \cdot 5)\sqrt{2} = 20\sqrt{2} + 10\sqrt{2} = 30\sqrt{2}$$

(c)

$$\frac{6 + \sqrt{44}}{2} =$$

**Solution:**

$$\frac{6 + \sqrt{44}}{2} = \frac{6 + \sqrt{4 \cdot 11}}{2} = \frac{6 + 2\sqrt{11}}{2} = 3 + \sqrt{11}$$

(d)

$$(\sqrt{3} - 7)(\sqrt{3} - 5) =$$

**Solution:**

$$(\sqrt{3} - 7)(\sqrt{3} - 5) = \sqrt{3}\sqrt{3} - 7\sqrt{3} - 5\sqrt{3} + 35 = 3 - 12\sqrt{3} + 35 = 38 - 12\sqrt{3}$$

(e)

$$(\sqrt{x} + 9)^2 =$$

**Solution:**

$$(\sqrt{x} + 9)^2 = (\sqrt{x} + 9)(\sqrt{x} + 9) = x + 9\sqrt{x} + 9\sqrt{x} + 81 = x + 18\sqrt{x} + 81$$

5. (5 points) Simplify the expressions by rationalizing the denominators, i.e., simplify the expression so that the denominator does not contain any radicals. (Hint: use the conjugate of the denominator.)

$$\frac{5}{\sqrt{x}+7}$$

**Solution:**
We multiply the numerator and denominator by the conjugate of the denominator, which is $\sqrt{x}-7$:

$$\frac{5}{\sqrt{x}+7}\cdot\frac{(\sqrt{x}-7)}{(\sqrt{x}-7)}=\frac{5\sqrt{x}-35}{x-49}$$