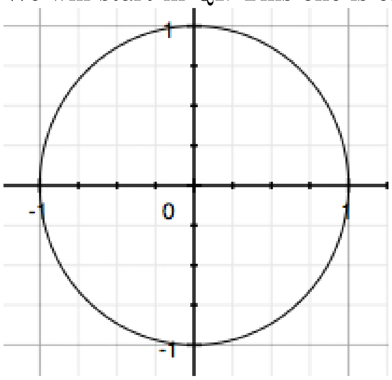
Solving basic cosine equations

Example 1: To solve $\cos(x) = \frac{1}{2}$ for x in the interval $[0, 2\pi)$.

We recall that $\cos(x)$ is the first coordinate of the point on the unit circle. Which quadrants have the first coordinate positive?
We will start in QI. This one is called the “principal solution” to the equation.



First solution:

- Find a point in QI whose first coordinate is $\frac{1}{2}$.
- Draw the QI reference triangle that goes with that point.
- Recognize this is one of our basic right triangles. Find the reference angle.

This gives us the first solution, called the “principal solution”. Write it down here:

Second solution: we can see that there is another point on the unit circle that also has first coordinate $\frac{1}{2}$ and it is in QIV.

- Reflect the first triangle into QIV.
- Find the coordinates of the reflected point on the unit circle.
- Find the reference angle for the reflected triangle.
- What’s the standard position angle (rotation) that goes with that point in QIV?

Now we have two solutions to $\cos(x) = \frac{1}{2}$ in $[0, 2\pi)$ (one full rotation) and we can see that these are the only ones. So the solutions are:

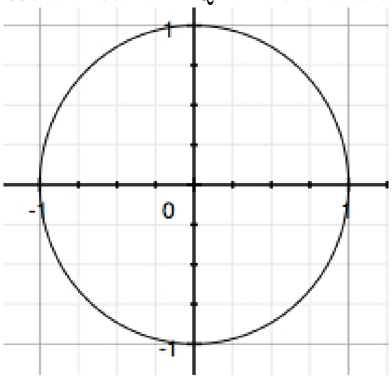
Example 2: find all the solutions of $\cos(x) = -\frac{\sqrt{2}}{2}$ in $[0, 2\pi)$
Which quadrants have the first coordinate negative?
We will start in QII. In case we are solving for a negative value of cosine, the solution in QII is called the “principal solution”.

Example 3: Find all the solutions of $\cos(x) = 0$ in $[0, 2\pi)$.

Example 4: Find all the solutions of $\cos(x) = 3$ in $[0, 2\pi)$

Solving basic sine equations

Example 1: To solve $\sin(x) = \frac{1}{2}$ for x in the interval $[0, 2\pi)$.
We recall that $\sin(x)$ is the second coordinate of the point on the unit circle.
Which quadrants have the second coordinate positive?
We will start in QI. This one is called the "principal solution" to the equation.



First solution:

- Find a point in QI whose second coordinate is $\frac{1}{2}$.
- Draw the QI reference triangle that goes with that point.
- Recognize this is one of our basic right triangles. Find the reference angle.

This gives us the first solution, called the "principal solution". Write it down here:

Second solution: we can see that there is another point on the unit circle that also has second coordinate $\frac{1}{2}$ and it is in QII.

- Reflect the first triangle into QII.
- Find the coordinates of the reflected point on the unit circle.
- Find the reference angle for the reflected triangle.
- What's the standard position angle (rotation) that goes with that point in QII?

Now we have two solutions to $\sin(x) = \frac{1}{2}$ in $[0, 2\pi)$ (one full rotation) and we can see that these are the only ones. So the solutions are:

Example 2: find all the solutions of $\sin(x) = -\frac{\sqrt{3}}{2}$ in $[0, 2\pi)$
Which quadrants have the second coordinate negative?
We will start in QIV. In case we are solving for a negative value of sine, the solution in QII is called the "principal solution".

Example 3: Find all the solutions of $\sin(x) = -1$ in $[0, 2\pi)$.

Example 4: Find all the solutions of $\sin(x) = 2$ in $[0, 2\pi)$