Mathematics 1575/D630, Spring 2015 **Exam #3**
Instructor: Suman Ganguli **May 13, 2015** **Name:** ______________________

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	10	10	10	10	10	10	10	10	10	10	100
Score:											

Show all your work and simplify your answers.

For #1-6, determine whether the infinite series converges or diverges. Justify your answer by using an appropriate test:

1. (10 points)

$$\sum_{n=1}^{\infty} \frac{2^n}{3^n}$$

Solution: Converges as geometric series with $r = 2/3 < 1$. Converges to

$$\frac{2/3}{1-2/3} = \frac{2/3}{1/3} = 2$$

2. (10 points)

$$\sum_{n=1}^{\infty} \frac{n^4}{10n^4 + n^2 + 1}$$

Solution: Diverges by the Divergence Test:

$$\lim_{n\to\infty} \frac{7n^4}{10n^4 + n^2 + 1} = \lim_{n\to\infty} \frac{7n^4}{n^4(10 + 1/n^2 + 1/n^4)} = \frac{7}{10} \neq 0$$

3. (10 points)

$$\sum_{n=1}^{\infty} n^{-0.05}$$

Solution: Diverges since it's a p-series with $p = 0.05 < 1$:

$$\sum_{n=1}^{\infty} n^{-0.05} = \sum_{n=1}^{\infty} \frac{1}{n^{0.05}}$$

4. (10 points)

$$\sum_{n=1}^{\infty} \frac{100^n}{n!}$$

Solution: Converges by the Ratio Test since $\rho = 0 < 1$:

$$\rho = \lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = \lim_{n\to\infty} \frac{100^{n+1}}{(n+1)!}\frac{n!}{100^n} = \lim_{n\to\infty} \frac{100}{n+1} = 0$$

5. (10 points)

$$\sum_{n=0}^{\infty} \frac{1}{\sqrt{9n^2+10}}$$

Solution: Diverges by Limit Comparison with $\sum \frac{1}{n}$:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1}{\sqrt{9n^2+10}}\frac{n}{1} = \lim_{n\to\infty} \frac{1}{\sqrt{n^2(9+10/n^2)}}\frac{n}{1} = \frac{1}{3}$$

6. (10 points)

$$\sum_{n=1}^{\infty} \frac{3^n}{n}$$

Solution: Diverges by Ratio Test the Ratio Test since $\rho = 3 > 1$:

$$\rho = \lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = \lim_{n\to\infty} \frac{3^{n+1}}{n+1}\frac{n}{3^n} = \lim_{n\to\infty} \frac{3n}{n+1} = 3$$

For #7-10, determine whether the alternating series is absolutely convergent, conditionally convergent, or divergent. Justify your answers:

7. (10 points)

$$\sum_{n=1}^{\infty} \frac{(-10)^n}{9^n}$$

Solution: Diverges as a geometric series with $r = -10/9$. Alternatively, by the Ratio Test since $\rho = 10/9 > 1$:

$$\rho = \lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = \lim_{n\to\infty} \frac{10^{n+1}}{9^{n+1}}\frac{9^n}{10^n} = \lim_{n\to\infty} \frac{10}{9} = \frac{10}{9}$$

8. (10 points)

$$\sum_{n=1}^{\infty} (-1)^n\, n^{-5}$$

Solution: Absolutely convergent since $\sum \frac{1}{n^5}$ converges as p-series with $p = 5 > 1$

9. (10 points)

$$\sum_{n=0}^{\infty}(-1)^n \frac{n^3+1}{n^7+1}$$

Solution: Absolutely convergent since $\sum \frac{n^3+1}{n^7+1}$ converges by Limit Comparison with $\sum \frac{1}{n^4}$:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^3+1}{n^7+1}\frac{n^4}{1} = \lim_{n\to\infty} \frac{n^3(1+1/n^3)}{n^7(1+1/n^7)}\frac{n^4}{1} = 1$$

10. (10 points)

$$\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt[3]{n}}$$

Solution: Conditionally convergent since $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$ diverges (p-series with $p = 1/3 < 1$) but $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt[3]{n}}$ converges by the Alternating Series Test: $a_n = \frac{1}{\sqrt[3]{n}}$ is a decreasing sequence and $\lim_{n\to\infty} \frac{1}{\sqrt[3]{n}} = 0$