Mathematics 1575/D630, Spring 2015 **Exam #1**
Instructor: Suman Ganguli **March 2, 2015** **Name:** ____________________

Run LaTeX again to produce the table

Show all your work and simplify your answers.

1. (20 points) Solve the following indefinite integrals:

(a)

$$\int \left(3x^6 - 2x^3 + \frac{1}{x} + \frac{1}{x^2}\right)\, dx =$$

Solution: $\dfrac{3x^7}{7} - \dfrac{x^4}{2} + \ln|x| - x^{-1} + C$

(b)

$$\int e^{\cos x} \sin x \; dx =$$

Solution: $u = \cos x, du = -\sin x\, dx \Longrightarrow -\displaystyle\int e^u du = -e^{\cos x} + C$

2. (20 points) Compute the values of the following definite integrals:

(a)

$$\int_1^{25} \frac{3\, dt}{\sqrt{t}} =$$

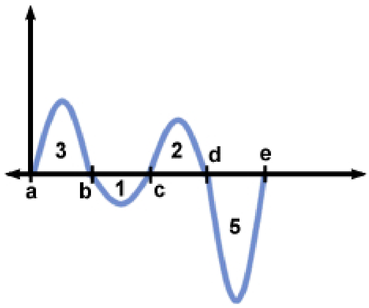
Solution: $3\displaystyle\int_1^{25} t^{-1/2} du = 3 * 2\left[t^{1/2}\right]_1^{25} = 6\left[25^{1/2} - 1\right] = 6(5-1) = 24$

(b)

$$\int_0^{\pi/4} (2 + \sin 4x)\; dx =$$

Solution: $\left[2x - \dfrac{1}{4}\cos 4x\right]_0^{\pi/4} = \left(\dfrac{\pi}{2} - \dfrac{1}{4}\cos\pi\right) - \left(0 - \dfrac{1}{4}\cos 0\right) = \left(\dfrac{\pi}{2} + \dfrac{1}{4}\right) - \left(0 - \dfrac{1}{4}\right) = \dfrac{1}{2} + \dfrac{\pi}{2}$

3. (20 points) This graph shows the areas of certain regions between a curve $y = f(x)$ and the x-axis. Use it to find the values of the following definite integrals:



(a) $\int_a^c f(x)\,dx =$

Solution: $3 - 1 = 2$

(b) $\int_b^c f(x)\,dx =$

Solution: -1

(c) $\int_a^e f(x)\,dx =$

Solution: $3 - 1 + 2 - 5 = -1$

(d) $\int_a^e |f(x)|\,dx =$

Solution: $3 + 1 + 2 + 5 = 11$

4. (20 points) Solve using the inverse trigonmetric integrals: $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$ and $\int \frac{du}{1+u^2} = \tan^{-1} u + C$

(a)

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx =$$

Solution: $u = \sin^{-1} x, du = \frac{dx}{\sqrt{1-x^2}} \implies \int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx = \int u du = \frac{u^2}{2} + C = \frac{(\sin^{-1} x)^2}{2} + C$

(b)

$$\int \frac{x\,dx}{1+x^4} =$$

Solution: $u = x^2, du = 2x\,dx \implies \int \frac{x\,dx}{1+x^4} = \frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}\tan^{-1} u + C = \frac{1}{2}\tan^{-1}(x^2) + C$

5. (10 points) Recall that the integration by parts formula is:

$$\int u\,dv = uv - \int v\,du \quad \text{or} \quad \int u(x)v'(x)\,dx = u(x)v(x) - \int v(x)u'(x)\,dx$$

Use integration by parts to solve:

$$\int xe^{3x}\,dx =$$

Solution: $u = x, dv = e^{3x} \Longrightarrow du = dx, v = \frac{1}{3}e^{3x}$

$$\int xe^{3x}\,dx = \frac{xe^{3x}}{3} - \frac{1}{3}\int e^{3x}\,dx = \frac{xe^{3x}}{3} - \frac{1}{9}e^{3x} + C$$

6. (10 points) Use the method of odd powers for trigonometric integrals to solve the integral below, i.e., in this case use the fact that $\cos^3\theta = \cos^2\theta\cos\theta = (1 - \sin^2\theta)\cos\theta$.

$$\int \sin^2\theta\cos^3\theta\,d\theta =$$

Solution: $\int \sin^2\theta\cos^3\theta\,d\theta = \int \sin^2\theta(1-\sin^2\theta)\cos\theta\,d\theta = \int(\sin^2\theta - \sin^4\theta)\cos\theta\,d\theta = \frac{\sin^3\theta}{3} - \frac{\sin^5\theta}{5} + C$