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Find the inverse of matrices.

1. A = 8(4) – 5(6) = 2

   $A^{-1} = \frac{1}{2}\begin{bmatrix} 4 & -6 \\ -5 & 8 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 5/2 & 4 \end{bmatrix}$

3. A = 7(-3) – 3(-6) = -3

   $A^{-1} = -\frac{1}{3}\begin{bmatrix} -3 & 3 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & -7/3 \end{bmatrix}$

5. Use the inverse found in Exercise 3 to solve the system

   $8x_1 + 6x_2 = 2$

   $5x_1 + 4x_2 = -1$

   x = A^{-1}b = $\begin{bmatrix} -3 & 3 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 7 \\ -9 \end{bmatrix}$

7. Let A = $\begin{bmatrix} 1 & 2 \\ 5 & 12 \end{bmatrix}$, $\boldsymbol{b}_1 = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, $\boldsymbol{b}_2 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$, $\boldsymbol{b} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$, $\boldsymbol{b}_4 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$.

   Find A^{-1}, and use it to solve the four equations

   A**x** = $\boldsymbol{b}_1$, A**x** = $\boldsymbol{b}_2$, A**x** = $\boldsymbol{b}_3$, A**x** = $\boldsymbol{b}_4$

   A = 1(12) – 2(5) = 2

   $A^{-1} = \frac{1}{2}\begin{bmatrix} 12 & -2 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -5/2 & 1/2 \end{bmatrix}$

   A**x** = $\boldsymbol{b}_1$

   $$\begin{bmatrix} 6 & -1 \\ -5/2 & 1/2 \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 6(-1) + (-1)3 \\ \left(-\frac{5}{2}\right)(-1) + \frac{1}{2}(3) \end{bmatrix} = \begin{bmatrix} -9 \\ 4 \end{bmatrix}$$

   A**x** = $\boldsymbol{b}_2$

   $$\begin{bmatrix} 6 & -1 \\ -5/2 & 1/2 \end{bmatrix}\begin{bmatrix} 1 \\ -5 \end{bmatrix} = \begin{bmatrix} 6(1) + (-1)(-5) \\ \left(-\frac{5}{2}\right)(1) + \frac{1}{2}(-5) \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \end{bmatrix}$$

   A**x** = $\boldsymbol{b}_3$

   $$\begin{bmatrix} 6 & -1 \\ -5/2 & 1/2 \end{bmatrix}\begin{bmatrix} 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 6(2) + (-1)6 \\ \left(-\frac{5}{2}\right)2 + \frac{1}{2}(6) \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$$

A**x** = $\boldsymbol{b}_4$

$$\begin{bmatrix} 6 & -1 \\ -5/2 & 1/2 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 6(3) + (-1)5 \\ \left(-\frac{5}{2}\right)3 + \frac{1}{2}(5) \end{bmatrix} = \begin{bmatrix} 13 \\ -5 \end{bmatrix}$$

9. a) True
   b)False because the inverse of AB is $B^{-1}A^{-1}$
   c)False, its $ad - bc \neq 0$
   d)True
   e)True

11. Let A be an invertible n x n matrix, and let B be an n x p matrix. Show that the equation Ax = B has a unique solution A^{-1}B.

    Replace x in Ax = B for A^{-1}B
    Ax = A (A^{-1}B) = $(AA^{-1})B = IB$ since a matrix multiplied by identity matrix is the matrix itself $IB = B$, therefore Ax = B

13. Suppose AB = AC, where B and C are n x p matrices and A is invertible. Show that B = C. Is this true, in general, when A is not invertible?

    Since A is invertible to show $B = C$ we can multiply the equation $AB = AC$ by A^{-1} where we get $A^{-1}AB = A^{-1}AC$
    $A^{-1}A = I,$ so $A^{-1}AB = A^{-1}AC$ can be rewritten as $IB = IC$ which is equivalent to $B = C$

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23. Suppose CA = I_n (the n x n identity matrix). Show that the equation A**x**= **0** has only the trivial solution. Explain why A cannot have more columns than rows.

We can multiply the vector **x** to CA = I_n
CA**x** = I_n**x** which can be rewritten as CA**x** = **x** because **x** was being multiplied by an identity matrix. Since A**x**= **0** we can substitute that in **x** = CA**x** and get C**0 = 0** which shows that it has only the trivial solution.
This also shows us that A is linearly independent because we only have the trivial solution therefore it cannot have more columns than rows.

25. Suppose A is an m x n matrix and there exist n x m matrices C and D such that CA =I_n and AD = I_m: Prove that m = n and C = D : [*Hint:* Think about the product CAD.]

We know that when A has the only trivial solution, the columns cannot be more than the rows and when A has solutions for every vector solution, the rows cannot be more than the columns. So in this case we can say that the columns equal the rows or m = n.
Since we are given a hint, the product CAD = C(AD) = CI_m = C
and also CAD = (CA)D = I_nD = D, therefore CAD = D = C.

27. Let u = $\begin{bmatrix} -3 \\ 2 \\ -5 \end{bmatrix}$ and v = $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Compute u^Tv, v^Tu, uv^T, vu^T

u^Tv = $[-3 \quad 2 \quad -5] * \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ = -3a+2b+5c

v^Tu = $[a \quad b \quad c] * \begin{bmatrix} -3 \\ 2 \\ -5 \end{bmatrix}$ = -3a+2b+5c

uv^T = $\begin{bmatrix} -3 \\ 2 \\ -5 \end{bmatrix} * [a \quad b \quad c]$ = $\begin{bmatrix} -3a & -3b & -3c \\ 2a & 2b & 2c \\ -5a & -5b & -5c \end{bmatrix}$

vu^T= $\begin{bmatrix} a \\ b \\ c \end{bmatrix} * [-3 \quad 2 \quad -5]$ = $\begin{bmatrix} -3a & 2a & -5a \\ -3b & 2b & -5b \\ -3c & 2c & -5c \end{bmatrix}$

33. Prove Theorem 3(d). [*Hint:* Consider the j^{th} row of$(AB)^T$]

Let A be an m x n matrix and B be an n x p matrix. Then, AB is an m x p matrix and $(AB)^T$ is an p x m matrix. B^T is an p x n matrix and A^T is an n x m matrix. Therefore $B^T A^T$ is an p x n matrix.