Homework

Section 1.9. page 78 Numbers 3-23 (odd)

3. $R^2 \rightarrow R^2$ Vertical shear that maps e_1 into e_1-3e_2 and leaves e_2 unchanged

$$A = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} [\text{e1} \quad e2]$$

5. $R^2 \rightarrow R^2$ Rotates counter-clockwise π/2.

$$A = \begin{bmatrix} cos\varphi & -sin\varphi \\ sin\varphi & cos\varphi \end{bmatrix}$$

When φ=π/2

7. $R^2 \rightarrow R^2$ Rotates through -3π/4 clockwise and reflects points through horizontal x_1-axis. {Hint: $T(e1) = (\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$}

$$A = [\begin{matrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{matrix}]$$

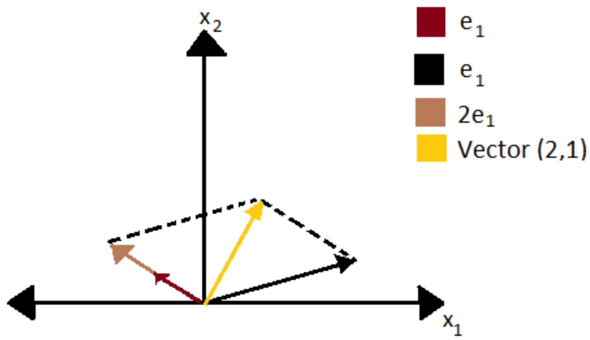
9. $R^2 \rightarrow R^2$ Reflects through x_1-axis and rotates points $\frac{-\pi}{2}$.

$$A = [\begin{matrix} \frac{-\pi}{2} & 0 \\ 0 & \frac{\pi}{2} \end{matrix}]$$

11. $R^2 \rightarrow R^2$ Reflects through x_1-axis then x_2-axis. Show T can also be described as linear transform that rotates about origin. What is the angle of rotation?

Angle of rotation $\frac{-2\pi}{3}$

13. $T:R^2 \rightarrow R^2$. $T(e_1)$ and $T(e_2)$ sketch vector (2,1).



15. Fill in:

$$\begin{bmatrix} 2 & -4 & 0 \\ & & x1 \\ 1 & 0 & -1][x2 \\ & & x3 \\ -1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 2x1 - 4x2 \\ x1 - x3 \\ -x2 + 3x3 \end{bmatrix}$$

17. $T(x_1,x_2,x_3)=(X_1-2x_2, 0, 2x_2+x_4,x_2-x_4)$

1 2 0 0 x1 x1+2x2

[0 0 0 0] = [x2] = [0]

0 2 0 0 x3 2x2+x4

0 1 0-1 x4 x2-x4

19. $T(x_1,x_2,x_3)=(x_1-5x_2+4x_3, x_2-6x_3)$

$$\begin{bmatrix}1 & -5 & 4 \\ 0 & 1 & -6\end{bmatrix}\begin{bmatrix}x1 \\ x2 \\ x3\end{bmatrix}=\begin{bmatrix}x1-5x2+4x3 \\ x2-6x3 \\ 0\end{bmatrix}$$

21. T: $R^2 \rightarrow R^2$. $T(x_1,x_2)=(x_1+x_2,4x_1+5x_2)$. Find T(x)=(3,8)

$3= x_1+x_2$ $8=4x_1+5x_2$

$X_1=7$ $x_2=-4$

$$T(x)=\begin{bmatrix}x1+x2 \\ 4x1+5x2\end{bmatrix}=\begin{bmatrix}1 & 1 \\ 4 & 5\end{bmatrix}\begin{bmatrix}7 \\ -4\end{bmatrix}$$

Augmented matrix: $\begin{bmatrix}1 & 1 & 3 \\ 4 & 5 & 8\end{bmatrix}$ Row reduce→ $\begin{bmatrix}1 & 0 & 7 \\ 0 & 1 & -4\end{bmatrix}$

23. A linear transform $T:R^n \rightarrow R^m$ is completely determined by its effect on the columns of the nxn identity matrix.

True (theorem 10)

If $T:R^2 \rightarrow R^2$ rotates vectors about the origin through an angle β, then T is a linear transformation.

True (example 3)

When two linear transformations are performed once on another, the combined effect may not always be a linear transformation.

False

A mapping T: $R^n \rightarrow R^m$ is into R^m if every vector R^n maps onto some vector in R^m.

False

If A is a 3x2 matrix, then the transformation x→ cannot be one-to-one

False