Linear Algebra

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23.

a. Every elementary row operation is reversible.
   True, because if you do the inverse of the initial operation your answer will be the original row.
b. A 5 X 6 matrix has six rows.
   False, this matrix has five rows and 6 columns.
c. The solution set of a linear system involving variables X_1,….,Xn is a list of numbers (S_1,….,Sn) that makes each equation in the system a true statement when the values S_1,….,Sn are substituted for X_1,….Xn, respectively.
   False, this statement only works for one given solution, not all given solutions.
d. Two fundamental questions about a linear system involve existence and uniqueness.
   True.

25.

$$\begin{bmatrix} 1 & 4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{bmatrix}$$

$X_1 - 4X_2 + 7X_3 = g$
$0 + 3X_2 – 5X_3 = h$
$-2x_1+5x_2-9x_3 = k$

2(Row 1) + (Row 3) = New Row 3
$2(X_1 - 4X_2 + 7X_3 = g) \Rightarrow 2X_1 - 8X_2+14X_3=2g$

$2X_1 - 8X_2+14X_3=2g$
<u>$+ -2x_1+5x_2-9x_3 = k$</u>
$0 - 3x_2+5x_3 = k+2g$ => New Row 3

$X_1 - 4X_2 + 7X_3 = g$
$0 + 3X_2 – 5X_3 = h$
$0 - 3x_2+5x_3 = k+2g$

Row 2 + Row 3 = New Row 3

$0 + 3X_2 – 5X_3 = h$
$+ 0 - 3x_2+5x_3 = k+2g$
$0 + 0 + 0 = h +k + 2g$ => New Row 3

$X_1 - 4X_2 +7X_3 = g$
$3X_2 – 5X_3 = h$
$0 + 0 + 0 = h +k + 2g$

$$\begin{bmatrix} 1 & 4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & 0 & 0 & h+k+2g \end{bmatrix}$$

27.

aX1 + bX2 = f
cX1 + dX2 = g

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} X1 \\ X2 \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{a} & \boldsymbol{b} & \boldsymbol{f} \\ \boldsymbol{c} & \boldsymbol{d} & \boldsymbol{g} \end{bmatrix}$$

Multiply Row 1 by (-c/a) + Row 2
-c/a(aX1 + bX2 = f)

(-ca/a)X1 + (-cb/a)X2 = (-cf/a)
+ cX1 + dX2 = g
0 + d-b(c/a) = g – f(c/a)

The system is consistent so there are solutions for X_1 and X_2.
d-b(c/a) ≠ 0

29. $\begin{bmatrix} 0 & -2 & 5 \\ 1 & 3 & -5 \\ 3 & -1 & 6 \end{bmatrix}$, $\begin{bmatrix} 3 & -1 & 6 \\ 1 & 3 & -5 \\ 0 & -2 & 5 \end{bmatrix}$

Step 1:

Row 1 and Row 3 were Swapped or Interchanged.

31. $\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 4 & -1 & 3 & -6 \end{bmatrix}$, $\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 0 & 7 & -1 & -6 \end{bmatrix}$

Step 2:

Row 3 was added to (-4)Row1 to get a new Row 3

Row 3 + (-4)Row 1 = New Row 3

New Row 1: $-4(X_1 - 2X_2 + X_3 = 0) = -4X_1 + 8X_2 - 4X_3 = 0$

$4X_1 - X_2 + 3X_3 = -6$
$+ -4X_1 + 8X_2 - 4X_3 = 0$
$-7X_2 - X_3 = -6$

New Matrix:

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 0 & 7 & -1 & -6 \end{bmatrix}$$

33.

$4T_4 - T_3 - 0T_2 - T_1 = 40$
$-T_4 + 4T_3 - T_2 - 0T_1 = 70$
$-0T_4 - T_3 + 4T_2 - T_1 = 60$
$-T_4 - 0T_3 - T_2 + 4T_1 = 30$

$$\begin{bmatrix} 4 & -1 & 0 & -1 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ -1 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} T4 \\ T3 \\ T2 \\ T1 \end{bmatrix} = \begin{bmatrix} 40 \\ 70 \\ 60 \\ 30 \end{bmatrix}$$

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9. Write a vector equation that is equivalent to the given system of equations.

$X_2 + 5X_3 = 0$
$4x_1 + 6X_2 - X_3 = 0$
$-x_1 \quad +3x_2 - 8x_3 = 0$

$$x1\begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} + x2\begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix} + x3\begin{bmatrix} 5 \\ -1 \\ -8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

11.

$$a1\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, a2\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, a3\begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}, b= \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$

Matrix Form:

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{bmatrix}$$

Step 1:

Swap Row 2 and Row 3

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 2 & 8 & 6 \\ -2 & 1 & -6 & -1 \end{bmatrix}$$

Step 2:

Multiply 2(Row 1) then add Row 3.

2(Row 1) + Row 3 = New Row 3

$X_1 + 0X_2 + 5X_3 = 2$
$0 + 2X_2 + 8X_3 = 6$
$-2x_1 + x_2 - 6x_3 = -1$

New Row 1: $2(X_1 + 0X2 +5X3 = 2) \Rightarrow 2X_1 + 0X2 +10X3 = 4$

$2X_1 + 0X_2 +10X_3 = 4$
$\underline{+-2x_1+x_2-6x_3 = -1}$
$0 \quad + X_2 +4X_3= 3$
New Matrix:

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 2 & 8 & 6 \\ 0 & 1 & 4 & 3 \end{bmatrix}$$

Step 3:

Row 2 + (-2)Row 3 = New Row 3

New Row 3: $-2(0 + X_2+ 4X_3 = 3) \Rightarrow 0 -2X_2 -8X_3 = -6$

$0 + 2X_2 + 8X_3 = 6$
$\underline{+0 -2X_2 -8X_3 = -6}$
$0 + 0 + 0 \quad = 0$
No, b = 0 so there is no linear combination for a1, a2, and a3.

13.

$$A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix}, \text{b=} \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$$

$X_1 - 4X_2 +2X_3 = 3$
$0 + 3X_2 + 5X_3 = -7$
$-2x_1+8x_2-4x_3 = -3$

Step 1:
Row 3 + 2(Row 1) = New Row 3

New Row 1: $2(X_1 - 4X_2 +2X_3 = 3) \Rightarrow 2X_1 - 8X_2 +4X_3 = 6$

$-2x_1+8x_2-4x_3 = -3$
$\underline{+2X_1 - 8X_2 +4X_3 = 6}$
$0 \quad + 0 \quad + 0 = 3$

$0 \neq 3$

There is no linear combination for b in the vectors formed from the columns of the matrix A.

15.

a1$\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$, a2$\begin{bmatrix} -5 \\ -8 \\ 2 \end{bmatrix}$, b= $\begin{bmatrix} 3 \\ -5 \\ h \end{bmatrix}$

For what value(s) of *h* is b in the plane spanned by a1 and a2?

X1$\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ + X2$\begin{bmatrix} -5 \\ -8 \\ 2 \end{bmatrix}$=$\begin{bmatrix} 3 \\ -5 \\ h \end{bmatrix}$

$X_1 - 5X_2 = 3$
$3X_1 - 8X_2 = 5$
$x_1 + 2x_2 = h$

$$\begin{bmatrix} 1 & -5 & 3 \\ 3 & -8 & -5 \\ -1 & 2 & h \end{bmatrix}$$

Step 1: -3(Row 1) + Row 2

New Row 1: $-3(X_1 - 5X_2 = 3) \Rightarrow -3X_1 + 15X_2 = -9$

$-3X_1 + 15X_2 = -9$
$\underline{+3X_1 - 8X_2 = 5}$
$+ 7X_2 = -14$: New Row 2

$$\begin{bmatrix} 1 & -5 & 3 \\ 0 & 7 & -14 \\ -1 & 2 & h \end{bmatrix}$$

Step 2:

Row 1 + Row 3 = New Row 3

$X_1 - 5X_2 = 3$
$\underline{+ -X_1 + 2X_2 = h}$
$0 \quad -3X_2 = 3 + h$

$$\begin{bmatrix} 1 & -5 & 3 \\ 0 & 7 & -14 \\ 0 & -3 & 3+h \end{bmatrix}$$

Step 3:

(1/2)Row 2 = New Row 2

$$\begin{bmatrix} 1 & -5 & 3 \\ 0 & 1 & -2 \\ 0 & -3 & 3+h \end{bmatrix}$$

Step 4:

3(Row 2) + Row 3 = New Row 3

$3(X_2 = -2) \Rightarrow 3X_1 + 3X_2 = -6$

$$\begin{array}{r} 3X_2 = -6 \\ \underline{+\ -3X_2 = 3+h} \\ 0 = h - 3 \end{array}$$:New Row 3

$$\begin{bmatrix} 1 & -5 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & h-3 \end{bmatrix}$$

The system has a solution to the augmented matrix when h-3=0.
b is in the plane spanned a1 and a2 when h=3