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Assignment 3

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In exercises 5-8, use the definition of *A***x** to write the matrix equation as a vector equation, or vice versa.

5) $\begin{bmatrix} 1 & 2 & -3 & 1 \\ -2 & -3 & 1 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$

$2\begin{bmatrix} 1 \\ -2 \end{bmatrix} + (-1)\begin{bmatrix} 2 \\ -3 \end{bmatrix} + 1\begin{bmatrix} -3 \\ 1 \end{bmatrix} + (-1)\begin{bmatrix} 1 \\ -1 \end{bmatrix} =$

$\begin{bmatrix} (2)+ & (-2)+ & (-3)+ & (-1) \\ (-4)+ & (3)+ & (1)+ & (1) \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$

7) $x_1\begin{bmatrix} 4 \\ -1 \\ 7 \\ -4 \end{bmatrix} + x_2\begin{bmatrix} -5 \\ 3 \\ -5 \\ 1 \end{bmatrix} + x_3\begin{bmatrix} 7 \\ -8 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$

$\begin{bmatrix} 4 & -5 & 7 \\ -1 & 3 & -8 \\ 7 & -5 & 0 \\ -4 & 1 & 21 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$

9) $5x_1 + x_2 - 3x_3 = 8$
$2x_2 + 4x_3 = 0$

$x_1\begin{bmatrix} 5 \\ 0 \end{bmatrix} + x_2\begin{bmatrix} 1 \\ 2 \end{bmatrix} + x_3\begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 5 & 1 & -3 \\ 0 & 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$

Given *A* and **b** in Exercises 11 and 12, write the augmented matrix for the linear system that corresponds to the matrix equation *A***x**=**b**. Then solve the system and write the solution as a vector.

11) $A = \begin{bmatrix} 1 & 3 & -4 \\ 1 & 5 & 2 \\ -3 & -7 & 6 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -2 \\ 4 \\ 12 \end{bmatrix}$

$$\begin{bmatrix} 1 & 3 & -4 & -2 \\ 1 & 5 & 2 & 4 \\ -3 & -7 & 6 & 12 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & -4 & -2 \\ 0 & 2 & 6 & 6 \\ 0 & 2 & -6 & 6 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 3 & -4 & -2 \\ 0 & 1 & 3 & 3 \\ 0 & 2 & -6 & 6 \end{bmatrix} * \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & -13 & -11 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 12 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -11 \\ 3 \\ 0 \end{bmatrix}$$

13) Let $\mathbf{u} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$ and $A = \begin{bmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{bmatrix}$. Is **u** in the plane in $\mathbb{R}^3$ spanned by the columns of *A*? Why or why not?

$$\begin{bmatrix} 3 & -5 & 0 \\ -2 & 6 & 4 \\ 1 & 1 & 4 \end{bmatrix} * \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 4 \\ -2 & 6 & 4 \\ 3 & -5 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 1 & 0 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & 8 & 12 \\ 0 & -8 & -12 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 4 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

Yes, **u** is in the plane $\mathbb{R}^3$ spanned by the columns of *A*, because every **u** in $\mathbb{R}^3$ is a linear combination of the columns of *A*. Also for each **u** in $\mathbb{R}^3$, the equation *A***x**=**u** has a solution.

15) Let $A = \begin{bmatrix} 3 & -1 \\ -9 & 3 \end{bmatrix}$ and **b** = $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$. Show that the equation *A***x**=**b** does not have a solution for all possible **b**, and describe the set of all **b** for which *A***x**=**b** *does* have a solution.

$$\begin{bmatrix} 3 & -1 & b_1 \\ -9 & 3 & b_2 \end{bmatrix} * \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \sim \begin{bmatrix} 3 & -1 & b_1 \\ 0 & 0 & 3b_1 + b_2 \end{bmatrix}$$

*A***x** doesn't have a solution for all possible **b** since 0$\neq 3b_1 + b_2$
In this case *A***x**=**b** is not consistent for all **b** because the echelon form of *A* has a row of zeros, and *A***x**=**b** has a solution if and only if **b** is a linear combination of all columns of *A*. If we had a pivot position in *A*, then *A***x**=**b** would have been consistent, and we would have a solution

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In Exercises 1-4, determine if the vectors are linearly independent. Justify each answer.

1) $\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -8 \end{bmatrix}$

$$\begin{bmatrix} 5 & 7 & 9 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & -6 & -8 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 3 & 1 \end{bmatrix} \sim \begin{bmatrix} 5 & 7 & 9 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 4 & 0 \end{bmatrix} * \begin{bmatrix} 1 & -7 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/4 \end{bmatrix} \sim$$

$$\begin{bmatrix} 5 & 0 & -5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} * \begin{bmatrix} 1/5 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Linearly Independent.
This equation has only one trivia solution, and all the columns, except the last, are a pivot position.

3) $\begin{bmatrix} 2 \\ -3 \end{bmatrix}, \begin{bmatrix} -4 \\ 6 \end{bmatrix}$

$$\begin{bmatrix} 2 & -4 & 0 \\ -3 & 6 & 0 \end{bmatrix} * \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 0 \\ -3 & 6 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Linearly Dependent.
The row operation on the associated augmented matrix shows that this equation has a nontrivial solution since we have a free variable.
Since x_2 is a free variable, it gives us many possible linear dependence, therefore, is not linearly independent