

Problems 1 & 3

Solve equation using elementary row operations on the equations or the augmented matrix. Follow systematic elimination procedure.

1.

$$x_1 + 5x_2 = 7$$

$$-2x_1 - 7x_2 = -5$$

$$\begin{bmatrix} 1 & 5 & 7 \\ -2 & -7 & 5 \end{bmatrix}$$

{Step 1}

$$2(R_1) + R_2 = R_2$$

$$\begin{bmatrix} 1 & 5 & 7 \\ 0 & 3 & 9 \end{bmatrix}$$

{Step 2}

$$1/3(R_2)$$

$$\begin{bmatrix} 1 & 5 & 7 \\ 0 & 1 & 3 \end{bmatrix}$$

$$x_2 = 3$$

{Step 3}

Substitute back in to get your  $x_1$  value.

$$x_1 + 5(3) = 7$$

$$x_1 + 15 = 7$$

$$-15 \quad -15$$

$$x_1 = -8$$

$$\text{Answer: } (x_1, x_2) = (-8, 3)$$

3.

$$x_1 + 2x_2 = 4$$

$$x_1 - x_2 = 1$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 1 & -1 & 1 \end{bmatrix}$$

{Step 1}

$$-1(R_1) + R_2 = R_2$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 3 & 3 \end{bmatrix}$$

{Step 2}

$$1/3(R_2)$$

$$\begin{bmatrix} 1 & 5 & 7 \\ 0 & 1 & 1 \end{bmatrix}$$

$$x_2 = 1$$

{Step 3}

Substitute  $x_2$  back in to get your  $x_1$  value.

$$x_1 + 2(1) = 4$$

$$x_1 + 2 = 4$$

$$-2 \quad -2$$

$$x_1 = 2$$

$$\text{Answer: } (x_1, x_2) = (2, 1)$$

- Be careful with calculations :D!

5.

State in words the next two elementary row operations.

$$\begin{bmatrix} 1 & -4 & -3 & 0 & 7 \\ 0 & 1 & 4 & 0 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -5 \end{bmatrix}$$

Step 1

Multiply  $R_2$  by -4 and add  $R_3$  which then equals your new  $R_2$ , then

Multiply  $R_3$  by 3 and add  $R_1$  and your result will be  $R_1$ .

Problems 7 & 9 have been augmented and reduced to row operations. In each case continue the appropriate row operations and describe solution set.

7.

$$\begin{bmatrix} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

There is no solution because in  $R_3$   $x_1, x_2, x_3$  carry a zero value and  $0 \neq 1$ .

9.

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -5 \\ 0 & 1 & -2 & 0 & -7 \\ 0 & 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

$x_4$  is given.

$$x_4 = 4$$

Step 1 Substitute  $x_4$  in  $R_3$  which will give you  $x_3$ .

$$x_3 = 14$$

Step 2 Substitute  $x_3$  in  $R_2$  which will give you  $x_2$ .

$$x_2 = 21$$

Step 3 Substitute  $x_2$  in  $R_1$  which will give you  $x_1$ .

$$x_1 = 16$$

Solve the systems in Exercises 11–14.

11.  $x_2 + 5x_3 = -4$

$$x_1 + 4x_2 + 3x_3 = -2$$

$$2x_1 + 7x_2 + x_3 = -2$$

Answer: Inconsistent

$$\begin{aligned}
 13. \quad x_1 & \quad - 3x_3 = 8 \\
 2x_1 + 2x_2 + 9x_3 & = 7 \\
 x_2 + 5x_3 & = -2
 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{bmatrix}$$

{Step 1}

$$-2(R_1) + R_2 = R_2$$

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{bmatrix}$$

{Step 2}

$$-2(R_3) + R_2 = R_2$$

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 0 & 5 & -5 \\ 0 & 1 & 5 & -2 \end{bmatrix}$$

{Step 3}

Interchange  $R_2$  with  $R_3$ .

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 5 & -5 \end{bmatrix}$$

{Step 4}

$$1/5(R_3) = R_3$$

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$x_3 = -1$$

{Step 4}

Substitute back in to get your  $x_2, x_1$ , values.

$$x_2 = 3$$

$$x_1 = 5$$

Determine if the systems in Exercises 15 and 16 are consistent. Do not completely solve the systems.

$$\begin{aligned} \mathbf{15.} \quad x_1 - 6x_2 &= 5 \\ x_2 - 4x_3 + x_4 &= 0 \\ -x_1 + 6x_2 + x_3 + 5x_4 &= 3 \\ -x_2 + 5x_3 + 4x_4 &= 0 \end{aligned}$$

Inconsistent there is no solution.

**17.** Do the three lines  $2x_1 + 3x_2 = -1$ ,  $6x_1 + 5x_2 = 0$ , and  $2x_1 - 5x_2 = 7$  have a common point of intersection? Explain.

Inconsistent

In Exercises 19–22, determine the value(s) of  $h$  such that the matrix is the augmented matrix of a consistent linear system.

$$\mathbf{19.} \quad \begin{bmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{bmatrix}$$

Answer:  $h \neq 2$

$$\mathbf{21.} \quad \begin{bmatrix} 1 & 4 & -2 \\ 3 & h & -6 \end{bmatrix}$$

All  $h$ .

For Questions 19 & 21 did not have a clue.