Section 1.3 Exercises

11. Given

$$a_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, a_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, a_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix} \& \ b = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$

To find out whether b is a linear combination of a_1, a_2 and a_3, we check if the vector equation $x_1a_1 + x_2a_2 + x_3a_3 = b$ had a solution or not.

$$x_1\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + x_2\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + x_3\begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix} \text{..................}(1)$$

Write the above equation in augmented matrix say M

$$\boldsymbol{M} = \begin{bmatrix} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{bmatrix}$$

Using $R_2 \rightarrow R_2 + 2R_1$

$$\boldsymbol{M} \sim \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 2 & 8 & 6 \end{bmatrix}$$

Using $R_3 \rightarrow R2_3 + 2R_2$

$$\boldsymbol{M} \sim \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The linear system corresponding to M has a solution, so the vector equation (1) has a solution and therefore b is a linear combination of $a_1, a_2,$ and a_3

13.

$$A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix}, b = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$$

Denote the columns of A by a_1, a_2 and a_3

$$\begin{matrix} & a_1 & a_2 & a_3 & b \\ & \downarrow & \downarrow & \downarrow & \downarrow \end{matrix}$$

$$[a_1\, a_2\, a_3\, b] = \begin{bmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & 0 & 3 \end{bmatrix}$$

Using $R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & 0 & 3 \end{bmatrix}$$

The system for this augmented matrix is inconsistent since $0 \neq 3$.

Hence b is not a linear combination of the columns of A.

Section 1.4 Exercises:

Q5.

Write a matrix equation as a vector equation using definition of AX.

By the definition of $A\cdot x$

$$2\begin{bmatrix}1\\-2\end{bmatrix}+(-1)\begin{bmatrix}2\\-3\end{bmatrix}+1\cdot\begin{bmatrix}-3\\1\end{bmatrix}+(-1)\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}-4\\1\end{bmatrix}$$

Therefore

$$2\begin{bmatrix}1\\-2\end{bmatrix}-1\begin{bmatrix}2\\-3\end{bmatrix}+1\cdot\begin{bmatrix}-3\\1\end{bmatrix}-1\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}-4\\1\end{bmatrix}$$

Q7.

$$x_1\begin{bmatrix}4\\-1\\7\\-4\end{bmatrix}+x_2\begin{bmatrix}-5\\3\\-5\\1\end{bmatrix}+x_3\begin{bmatrix}7\\-8\\0\\2\end{bmatrix}=\begin{bmatrix}6\\-8\\0\\-7\end{bmatrix}$$

The left side of the equation is linear combination of three vectors. Given system of equations is equivalent to a single matrix equation $AX=B$

Where $\boldsymbol{A} = \begin{bmatrix} 4 & -5 & 7 \\ -1 & 3 & -8 \\ 7 & -5 & 0 \\ -4 & 1 & 2 \end{bmatrix}$, $\boldsymbol{X} = \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$

Thus

$\boldsymbol{AX = B}$

$$\Rightarrow \begin{bmatrix} 4 & -5 & 7 \\ -1 & 3 & -8 \\ 7 & -5 & 0 \\ -4 & 1 & 2 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$$

Q9.

$5x_1 + x_2 - 3x_3 = 8$

$2x_2 + 4x_3 = 0$

$$x_1\begin{bmatrix} 5 \\ 0 \end{bmatrix} + x_2\begin{bmatrix} 1 \\ -2 \end{bmatrix} + x_3\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 1 & 3 \\ 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$$