

Section 1.4 Exercises

Q5. Write the matrix equation as a vector equation using definition of Ax .

$$\begin{bmatrix} 1 & 2 & -3 & 1 \\ -2 & -3 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$$

By the definition of Ax

$$2 \begin{bmatrix} 1 \\ -2 \end{bmatrix} + (-1) \cdot \begin{bmatrix} 2 \\ -3 \end{bmatrix} + 1 \cdot \begin{bmatrix} -3 \\ 1 \end{bmatrix} + (-1) \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$$

Therefore

$$2 \begin{bmatrix} 1 \\ -2 \end{bmatrix} - 1 \cdot \begin{bmatrix} 2 \\ -3 \end{bmatrix} + 1 \cdot \begin{bmatrix} -3 \\ 1 \end{bmatrix} - 1 \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$$

Q7.

$$x_1 \begin{bmatrix} 4 \\ -1 \\ 7 \\ -4 \end{bmatrix} + x_2 \begin{bmatrix} -5 \\ 3 \\ -5 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ -8 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$$

The left side of the equation is linear combination of three vectors. Given system of equations is equivalent to an angle matrix equation $AX = B$

$$\text{Where } A = \begin{bmatrix} 4 & -5 & 7 \\ -1 & 3 & -8 \\ 7 & -5 & 0 \\ -4 & 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, B = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$$

Thus
 $AX=B$

$$\begin{bmatrix} 4 & -5 & 7 \\ -1 & 3 & -8 \\ 7 & -5 & 0 \\ -4 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$$

Q9.

$$\begin{aligned} 5x_1 + x_2 - 3x_3 &= 8 \\ 2x_2 + 4x_3 &= 0 \end{aligned}$$

$$x_1 \begin{bmatrix} 5 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 1 & -3 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$$