To the Pre-Service Teacher

In one sense, these three volumes are just textbooks, and you may feel you have gone through too many textbooks in your life to need any fresh advice. Nevertheless, we are going to suggest that you approach these volumes with a different mindset than what you may have used with other textbooks, because you will soon be using the knowledge you gain from these volumes to teach your students. Reading other textbooks, you would likely congratulate yourself if you could achieve mastery over 90% of the material. That would normally guarantee an A. More is at stake with these volumes, however, because they directly address what you will need to know in order to write your lessons. Ask yourself whether a *mathematics* teacher whose lessons are correct only 90% of the time should be considered a good teacher. To be blunt, such a teacher would be a near disaster. So your mission in reading these volumes should be to achieve nothing short of total mastery. **You are expected to know this material 100%**. To the extent that the content of these three volumes is just K–12 mathematics, this is an achievable goal. This is the standard you have to set for yourself. Having said that, we also note explicitly that many *Mathematical Asides* are sprinkled all through the text, sometimes in the form of footnotes. These are comments—usually from an advanced mathematical perspective—that try to shed light on the mathematics under discussion. *The above reference to "total mastery" does not include these comments.*

You should approach these volumes differently in yet another respect. Students' typical attitude towards a math course is that if they can do all the homework problems, then most of their work is done. Think back on your calculus courses or any of the math courses when you were in school, and you will understand how true this is. But since these volumes are designed specifically for teachers, your emphasis cannot be limited to merely doing the homework assignments because your job will be more than just helping students to do homework problems. When you stand in front of a class, what you will be talking about, *most of the time*, will not be the exercises at the end of each section but the concepts and skills in the exposition proper.[1] For example, very likely you will soon have to convince a class on geometry why the Pythagorean theorem is correct. There are two proofs of this theorem in these volumes, one in Chapter 5 of this volume and the other in Chapter 4 of [Wu2020c]. Yet on neither occasion is it possible to assign a problem that asks for a proof of this theorem. There are problems that can assess whether you know

[1] I will be realistic and acknowledge that there *are* teachers who use class time only to drill students on how to get the right answers to exercises, often without reasoning. But one of the missions of these three volumes is to steer you away from that kind of teaching. See **To the Instructor** on pp. xxvii ff.

enough about the Pythagorean theorem to apply it, but how do you assess whether you know *how to prove the theorem* when the proofs have already been given in the text? It is therefore entirely up to you to achieve mastery of everything in the text itself. One way to check is to pick a theorem at random and ask yourself: Can I prove it without looking at the book? Can I explain its significance? Can I convince someone else why it is worth knowing? Can I give an intuitive summary of the proof? These are questions that you will have to answer as a teacher. To the extent possible, these volumes try to provide information that will help you answer questions of this kind. I may add that the most taxing part of writing these volumes was in fact to do it in a way that would allow you, as much as possible, to adapt them for use in a school classroom with minimal changes. (Compare, for example, **To the Instructor** on pp. xxvii ff.)

There is another special feature of these volumes that I would like to bring to your attention: these volumes are essentially *school textbooks written for teachers*, and as such, you should read them with the eyes of a school student. When you read Chapter 1 of this volume on fractions, for instance, picture yourself in a sixth-grade classroom and therefore, no matter how much abstract algebra you may know or how well you can explain the construction of the quotient field of an integral domain, you have to be able to give explanations in the language of sixth-grade mathematics (i.e., to sixth graders). Similarly, when you come to Chapter 6, you are developing algebra from the beginning, so even the use of symbols will be an issue (it is in fact the *key* issue; see Section 6.1 on pp. 298 ff.). Therefore, be very deliberate and explicit when you introduce a symbol, at least for a while.

The major conclusions in these volumes, as in all mathematics books, are summarized into **theorems**. Depending on the author's (and other mathematicians') whims, theorems are sometimes called **propositions**, **lemmas**, or **corollaries** as a way of indicating which theorems are deemed more important than others. Roughly speaking, a proposition is not regarded to be as important as a theorem, a lemma is conceptually less important than a proposition, and a corollary is supposed to follow immediately from the theorem or proposition to which it is attached. (Incidentally, a formula or an algorithm is just a theorem.) This idiosyncratic classification of theorems started with Euclid around 300 BC, and it is too late to do anything about it now. The main concepts of mathematics are codified into **definitions**. Definitions are set in **boldface** in these volumes when they appear for the first time; a few truly basic ones are even individually displayed in a separate paragraph, but most of the definitions are embedded in the text itself, so you should watch out for them.

The statements of the theorems, and especially their proofs, depend on the definitions, and proofs are the guts of mathematics.

Please note that when I said above that I expect you to know everything in these volumes, I was using the word *"know"* in the way mathematicians normally use the word. They do not use it to mean simply "know the statement by heart". Rather, *to know* a theorem, for instance, means *know the statement by heart, know its proof, know why it is worth knowing, know what its potential implications are, and finally, know how to apply it in new situations.* If you know anything short of this, how can you expect to be able to answer your students' questions? At the very least, you should know by heart all the theorems and definitions as well as the main ideas of each proof because, if you do not, it will be futile to talk about the other aspects of knowing. Therefore, a preliminary suggestion to help you master the content of these volumes is for you to

> copy out the statements of every definition, theorem, proposition, lemma, and corollary, along with page references so that they can be examined in detail when necessary,

and also to

> form the habit of summarizing the main idea(s) of each proof.

These are good study habits. When it is your turn to teach your students, be sure to pass along these suggestions to them.

You should also be aware that reading a mathematics book is not the same as reading a gossip magazine. You can probably flip through one of the latter in an hour or less. But in these volumes, there will be many passages that require slow reading and re-reading, perhaps many times. I cannot single out those passages for you because they will be different for different people. We do not all learn the same way. What you can take for granted, however, is that mathematics books make for exceedingly slow reading. (Nothing good comes easy.) Therefore if you get stuck, time and time again, on a sentence or two in these volumes, take heart, because this is the norm in mathematics learning.