Pedagogical Comments. We wish to address the issue of why TSM's standard formula for the addition of fractions in terms of their *least common denominator* is pedagogically unsound. Given $\frac{k}{\ell}$ and $\frac{m}{n}$, students are told to add $\frac{k}{\ell} + \frac{m}{n}$ by first finding the least common denominator of the fractions, which is by definition the LCM[32] of the denominators ℓ and n, say B. Then if $B = \ell n' = \ell' n$ for some whole numbers ℓ' and n', the sum of these two fractions—according to TSM—is given by

$$\frac{k}{\ell} + \frac{m}{n} = \frac{kn'}{\ell n'} + \frac{\ell' m}{\ell' n} = \frac{kn' + \ell' m}{B}.$$

First of all, TSM usually offers this formula as a definition of the sum of fractions, and this is certainly unacceptable because it bears no resemblance to the intuitive notion of addition as "combining things". But even as a formula for addition, it is no less objectionable because, as we have seen in (1.12) on page 33, it is not necessary to use the LCM of ℓ and n when the product of the denominators, ℓn, is both adequate and more natural as a common denominator. Finally, this formula is destructive in terms of mathematics learning because many students confuse the LCM of two whole numbers with their *greatest common divisor* (see page 138 for this concept) and there is no reason to complicate the learning of a simple skill by artificially inflating its complexity.

The same comment applies to the use of least common denominator for the subtraction of fractions.

[32] Least common multiple. For a precise definition, see Exercise 4 on page 156.

Be sure to take every opportunity to eradicate this approach to the addition and subtraction of fractions from your teaching, because it has caused great harm to mathematics learning. **End of Pedagogical Comments.**

Mathematical Aside: The use of the least common denominator for the definition of the addition of fractions is more than just a pedagogical disaster. From a *mathematical* perspective, it is conceptually incorrect. If it were necessary to find the LCM of the two denominators before the addition of two fractions could be defined, it would imply that addition cannot be performed in the field of quotients of an integral domain unless the latter has the special property that any two elements in it have an LCM. This is almost the statement that addition cannot be defined in the field of quotients of an integral domain unless the domain has the *unique factorization property*. However, we know that this is false because addition can be defined in the field of quotients of *any* domain.