Pedagogical Comments. It does not seem likely, as of 2020, that a *proof* of any of the following theorems will ever make its way into the K–12 classroom: Theorem 3.1 (page 139) on the existence of a unique reduced form of a fraction, Theorem 3.2 (page 140) on the Euclidean algorithm, Theorem 3.6 (page 149) on the fundamental theorem of arithmetic, Theorem 3.8 (page 152) on the characterization of fractions which are finite decimals, and finally, Theorem 3.9 (page 153) on square roots that are not rational. This chapter therefore only serves the purpose of enriching your mathematical culture but does not belong to your minimal survival kit for teaching, *or so it would seem.*

The reality is a little different, however. Without having gone through these proofs, can a teacher convey to students with conviction that division-with-remainder is not the mindless rote skill that TSM makes it out to be but is, rather, a powerful mathematical tool that connects the reduced form of a fraction to the

unique prime decomposition of a positive integer? Without having gone through the proofs of Theorem 3.8 and Theorem 3.9 with care, can a teacher learn not to spread TSM's misinformation that it is the *existence* of the prime decomposition of a positive integer that matters but that there is no need to discuss its *uniqueness*? Without having gone through the proof of Theorem 3.1, won't teachers continue to insist that only fractions in lowest terms will be accepted as correct answers because—as TSM would have it—getting to the reduced form of a fraction is so easy? How can teachers avoid this pitfall if they have never faced a fraction such as $\frac{899}{1147}$, or even just something as simple as $\frac{143}{91}$?

A math teacher's mathematical content knowledge therefore cannot be circumscribed, literally, by the topics in the school mathematics curriculum. At a time when school mathematics education must rid itself of TSM, teachers also need to know some of the mathematical ideas underlying the curriculum itself to better understand why change is necessary. This chapter was designed to contribute toward fulfilling this need. It is for this reason that we consider the proofs of these theorems to be a vital part of the basic content knowledge of mathematics teachers. **End of Pedagogical Comments.**