Suppose that we use Euler's method to approximate the solution to the differential equation

$$\frac{dy}{dx} = \frac{x^1}{y}; \qquad y(0.5) = 6.$$

Let $f(x, y) = x^1/y$.
We let $x_0 = 0.5$ and $y_0 = 6$ and pick a step size $h = 0.2$. Euler's method is the the following algorithm. From $x_n$ and $y_n$, our approximations to the solution of the differential equation at the nth stage, we find the next stage by computing

$$x_{n+1} = x_n + h, \qquad y_{n+1} = y_n + h \cdot f(x_n, y_n).$$

Complete the following table. Your answers should be accurate to at least seven decimal places.

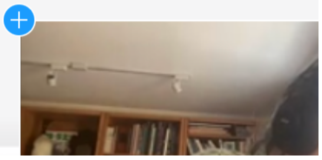
| $n$ | $x_n$ | $y_n$ |
|---|---|---|
| 0 | 0.5 | 6 |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

The exact solution can also be found using separation of variables. It is

$y(x) =$ $\boxed{y = \sqrt{x^2 + 35.75}}$

Thus the actual value of the function at the point $x = 1.5$

$y(1.5) =$ . $\boxed{6.164414}$



$y' = \frac{x}{y}$ $\quad y(0.5) = 6,$ $\quad h = 0.2$

| $n$ | $h$ | $x_n$ | $y_n$ | $K$ | $y_{n+1}$ |
|---|---|---|---|---|---|
| 0 | 0.2 | 0.5 | 6 | 0.083333333 | 6.01666666667 |
| 1 | 0.2 | 0.7 | 6.01666666667 | 0.11634349 | 6.03993536 |
| 2 | 0.2 | 0.9 | 6.03993536 | continue | |
| 3 | 0.2 | 1.1 | | | |
| 4 | 0.2 | 1.3 | | | |
| 5 | 0.2 | 1.5 | | | |

Round 1:

$f(x,y) = \frac{x}{y}, \quad x = 0.5, \quad y = 6$

$K = f(0.5, 6) = \frac{0.5}{6} = 0.083333333$

$y_{n+1} = 6 + (.08\bar{3})(0.2) = 6.01666666667$

Round 2

$K = f(.7, 6.01\bar{6}) = \frac{.7}{6.01\bar{6}} = 0.11634349$

$y_2 = 6.01666666667 + 0.11634349(0.2)$

$y_2 = 6.03993536$

Solve

$$\frac{dy}{dx} = \frac{x^1}{y}; \qquad y(0.5) = 6.$$

$$y' = \frac{x}{y}$$

$$\int y \cdot y' \, d\cancel{x} = \int x \, d\cancel{x}$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + c$$

$$y^2 = x^2 + 2c$$

$$y = \pm\sqrt{x^2 + 2c}$$

$y(.5) = 6$

choose "+" positive branch since $y=6$ is positive.

$$6 = \sqrt{(.5)^2 + 2c}$$

$$6 = \sqrt{.25 + 2c}$$

$$36 = .25 + 2c$$
$$-.25 \quad -.25$$

$$35.75 = 2c$$

$$c = \frac{35.75}{2} = 17.875$$

$$y = \sqrt{x^2 + 2(17.875)}$$

$$\boxed{y = \sqrt{x^2 + 35.75}}$$

$$y = (x^2 + 35.75)^{1(1/2)}$$

actual value $y(1.5) = \sqrt{(1.5)^2 + 35.75}$

$$= \boxed{6.164414}$$

# WeBWork Problem 3

Euler's method to estimate $y(1.4)$

$y' = x - xy$, $y(1) = 2$

## 1. $h = 0.2$

| $i$ | $h$ | $x_i$ | $y_i$ | $K$ | $y_{i+1}$ |
|---|---|---|---|---|---|
| 0 | 0.2 | 1 | 2 | | |
| 1 | 0.2 | 1.2 | | | |
| 2 | 0.2 | 1.4 | | | |

## 2. $h = 0.1$

$y' = x - xy$, $y(1) = 2$, find $y(1.4)$

| $i$ | $h$ | $x_i$ | $y_i$ | $K$ | $y_{i+1}$ |
|---|---|---|---|---|---|
| 0 | 0.1 | 1 | 2 | | |
| 1 | 0.1 | 1.1 | | | |

| | | |
|---|---|---|
| 2 | 0.1 | 1.2 |
| 3 | 0.1 | 1.3 |
| 4 | 0.1 | 1.4 |