Example 1: Suppose y(x) is a solution to the initial value problem dy/dx=3-2x-0.5y, y(1)=0.6. Find an approximate value of y(2) using Euler's method with step size h=0.5.

$y' = 3 - 2x - 0.5y \quad , \quad y(1) = 0.6$

find $y(2)$

i	h	x_i	y_i	K	y_{i+1}
0	0.5	1	0.6	0.7	
1	0.5	1.5			
2	0.5	2			

$y'(1, 0.6) = 3 - 2(1) - 0.5(0.6)$

$y_1 = y_0 + Kh = 0.6 + 0.7(0.5) =$

Example 1: Suppose y(x) is a solution to the initial value problem dy/dx=3-2x-0.5y, y(1)=0.6. Find an approximate value of y(2) using Euler's method with step size h=0.5.

NOTE: The solution to this initial value problem is:

y(x) = -4 x - 15.498 e^(-0.5 x) + 14

"CORRECT" ANS: y(2) = 0.298612

i	h	x_i	y_i	k = f(x_i,y_i)	y_(i+1)
0	0.5	1.0	0.6	0.7	0.95
1	0.5	1.5	0.95	-0.475	0.7125
2	0.3	2.0	**0.7125**		

$y' = \boxed{3 - 2x - 0.5y} = f(x,y)$

Example 1: Suppose y(x) is a solution to the initial value problem dy/dx=3-2x-0.5y, y(1)=0.6. Find an approximate value of y(2) using Euler's method with step size h=0.5.

Given a point (x_i, y_i), how do we Find/compute the next point?

- Find $x_{i+1} = x_i + h$
- Find $K_1 = f(x_i, y_i)$
- find $z_i = y_i + K_1 \cdot h$
- find $K_2 = f(x_{i+1}, z_i)$
- Find $y_{i+1} = y_i + \frac{K_1 + K_2}{2} \cdot h$

$h = 0.5$

$y(2) = ?$

$y' = 3 - 2x - 0.5y \quad y(1) = 0.6$

i	h	x_i	y_i	K_1	Z_1	K_2	y_{i+1}
0	.5						
1	.5						
2	.5						

IMPROVED EULER METHOD
Given a point (x_i,y_i), how do we find the next point, (x_(i+1),y_(i+1))
Calculate:

- Find x_(i+1) = x_i + h
- Find k1=f(t_i, y_i)
- Find z_(i+1) = y_i + h*k1
- Find k2 = f(t_(i+1), z_(i+1))
- Find y_(i+1)=y_i+h * [k1+k2]/2
- Now we have (x_(i+1),y_(i+1))

Example 2: Suppose y(x) is a solution to the initial value problem dy/dx=3-2x-0.5y, y(1)=0.6. Find an approximate value of y(2) using the Improved Euler's method with step size h=0.5.

"CORRECT" ANS: y(2) = 0.298612

i	h	x_i	y_i	k1	z_(i+1)	k2	y_(i+1)
0	0.5	1	0.6	0.7	0.95	-0.475	0.65625
1	0.5	1.5	0.65625	-0.328125	0.4921875	-1.24609375	0.2626953125
2	0.5	2	**0.2626953125**				

ROUND 1:
k1 = f(1,0.6)=3-2x-0.5y = 3-2(1)-0.5(0.6) =
z = y_i + k1*h=0.6+0.7*0.5 = 0.95 ← this is a temporary y-value
k2 = f(1.5,0.95)=3-2(1.5)-0.5(.95) = -0.475 ← slope at right side of interval
y_(i+1)=y_i + (k1+k2)/2 * h = 0.6 + (0.7 -0.475)/2 *(0.5) = 0.65625

ROUND 2:
k1 = f(1.5,0.65625) = 3-2(1.5)-0.5(.65625) = -0.328125
z = 1.5 + -0.328125 * (0.5) = 0.4921875
k2 = f(2,0.4921875) = 3-2(2)-0.5(0.4921875)=-1.24609375
y_(i+i)=0.65625 + (-0.328125+-1.24609375)/2 *(0.5)= 0.2626953125

ANS: according to Improved Euler's Method, **y(2) == 0.2626953125**

COMPARE:
Euler's: **y(2)==0.7125**
Improved Euler's: **y(2) == 0.2626953125**
Actual Value: ***y(2) = 0.298612***