A thermometer is taken from a room where the temperature is 25^oC to the outdoors, where the temperature is -6^oC. After one minute the thermometer reads 11^oC.

(a) What will the reading on the thermometer be after 4 more minutes?

☐ ,

(b) When will the thermometer read -5^oC?

☐ minutes after it was taken to the outdoors.

Solution:

Find $T(t) =$ ________

Newton's Law of Cooling

$$T' = -k(T - T_m)$$

$$T_m = -6$$

what is $25°C$? initial temperature when $t=0$, $T = 25$

$\boxed{T(0) = 25}$ ← initial condition.

when $t = 1$ min, temperature $T = 11$

$\boxed{T(1) = 11}$

$$T' = -k(T - (-6))$$

$$T' = -kT - 6k \longrightarrow T' + kT = -6k$$

1st order linear.

STEP 1 single solution T_1 to complementary:

$$T' + kT = 0$$

$$\frac{T'}{T} = \frac{-kT}{T}$$

$$\int \frac{T'}{T} dt = \int -k \, dt$$

$$\ln|T| = -kt + C$$

choose $C = 0$.

$$e^{\ln|T|} = e^{-kt}$$

$$|T| = e^{-kt}$$

$$T = \pm e^{-kt} \quad \text{choose } +.$$

$$T_1 = e^{-kt}$$

STEP 2 guess $T = u \cdot T_1$

$$T = ue^{-kt}$$

$$T' = -kue^{-kt} + u'e^{-kt}$$

sub $T' + kT = -6k$

$$\cancel{-kue^{-kt}} + u'e^{-kt} + \cancel{kue^{-kt}} = -6k$$

$$u'e^{-kt} = -6k$$

$$\int u'\,dt = \int -6ke^{kt}\,dt$$

$$u = -6\cancel{k}\,\frac{e^{kt}}{\cancel{k}} + C$$

$$\boxed{u = -6e^{kt} + C}$$

$$T = (-6e^{kt} + C)e^{-kt}$$

$$\boxed{T = -6 + Ce^{-kt}}$$

Initial values

$T(0)=25$

$T(1)=11$

$$25=-6+Ce^{-k\cdot 0}$$

$$25=-6+C$$

$$\boxed{31=C}$$

$$11=-6+Ce^{-k\cdot 1}$$

$$11=-6+31e^{-k}$$

solve for k:

$+6 \quad +6$

$$\frac{17}{31}=\frac{31e^{-k}}{31}$$

$$\frac{17}{31}=e^{-k}$$

$$\ln\left(\frac{17}{31}\right)=-k$$

$$\boxed{k=-\ln\left(\frac{17}{31}\right)}$$

$$\boxed{k=0.60077}$$

temperature of thermometer

over time t:

$$\boxed{T = -6 + 31e^{-.60077t}}$$

one minute the thermometer reads 11°C.

(a) What will the reading on the thermometer be after 4 more minutes?

☐ ,

(b) When will the thermometer read -5^oC?

☐ minutes after it was taken to the outdoors.

Solution:

a) Find T when $t = 5$ min $\left(\begin{matrix}1\text{ min} + \\ 4\text{ more}\end{matrix}\right)$

$$T = -6 + 31e^{-.60077(5)} = -6 + 31e^{(-\ln(17/31))(5)}$$

$$\boxed{T \approx -4.46253\,°C}$$

b) find t when $T = -5°C$

$$-5 = -6 + 31e^{-.60077t}$$

$$+6 \qquad +6$$

$$\frac{1}{31} = \frac{31e^{-.60077t}}{31}$$

$(.60077t$

$$\frac{1}{31} = e^{-.60077t}$$

$$\ln\left(\frac{1}{31}\right) = -.60077t$$

$$\frac{\ln(1/31)}{-.60077} = t$$

$$\boxed{t = 5.71598 \text{ minutes}}$$

A species of rabbits has a growth rate of 0.625 / month. If a population of foxes inhabits the same forest and kills 25 rabbits per day, find the general solution describing the population of rabbits.

P(t)= * Assume that one month = 30 days.

Hint:

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P = population

t = time in months

$$\boxed{P' = rP - K}$$

$r = 0.625$

$K = \#$ rabbits killed each month $= 25(30)$

$K = 750$

$$\boxed{p' = 0.625p - 750}$$

$$\boxed{p' - 0.625p = -750}$$ * (linear 1^{st} order)

solve to find $P(t) =$ ________ ?

We don't know any initial condition (# of rabbits at a given time), so our answer for $p(t)$ will have a ~~[illegible]~~ constant C in it

$$\boxed{p(t) = \underline{\hspace{6cm}}}$$