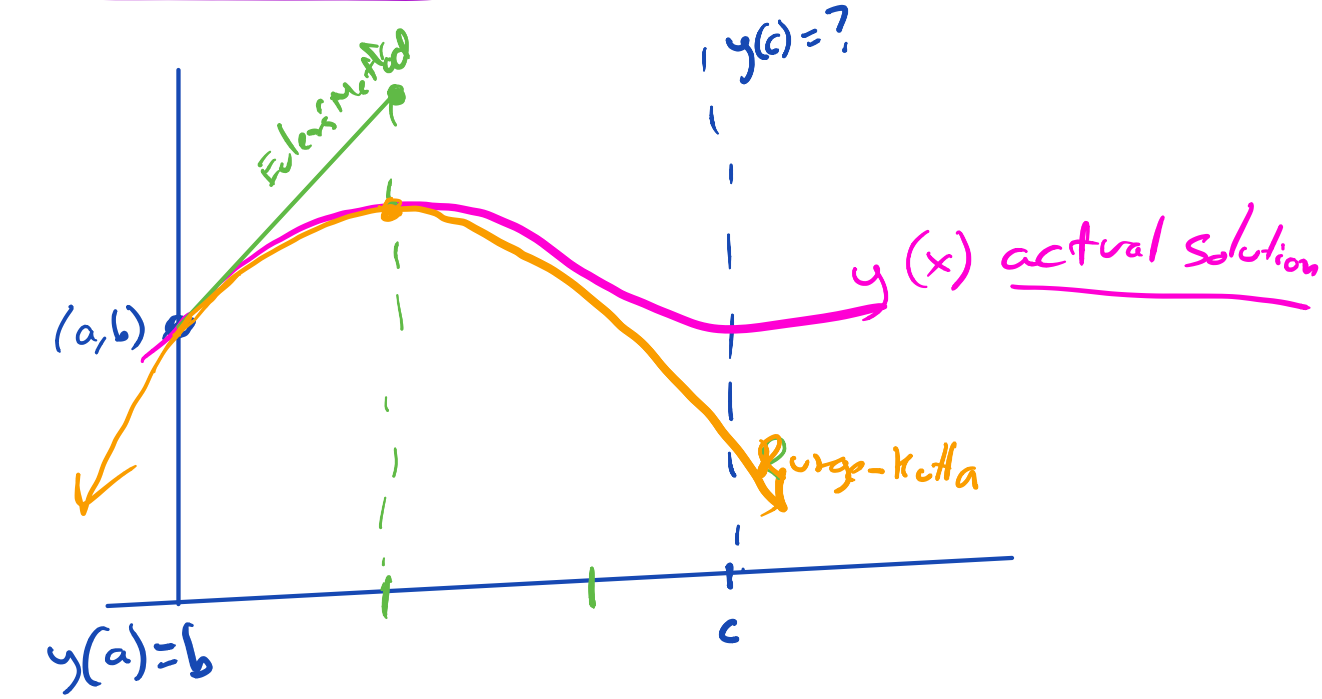
Numerical Methods:

Eulers Method: From a point (x_i, y_i), we estimate the next point (x_{i+1}, y_{i+1}) by using a straight line w/ slope at (x_i, y_i).

Improved Euler's Method: from a point (x_i, y_i) estimate next point (x_{i+1}, y_{i+1}) by using a straight line w/ slope averaging the slope at (x_i, y_i) and the rightside of the interval (x_{i+1}, z)

BIG IDEA

Runge-Kutta: from a point (x_i, y_i), estimate next point (x_{i+1}, y_{i+1}) by using a parabola.



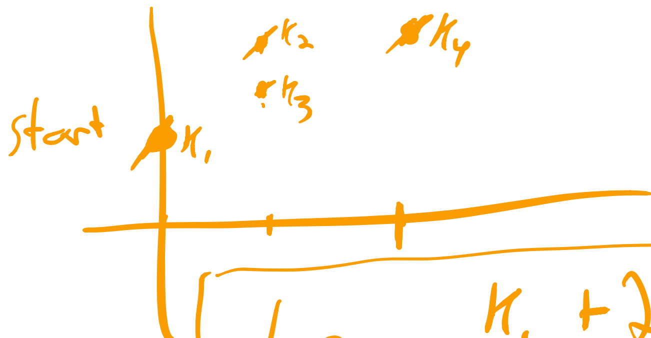
Fact about parabolas on interval $[d, e]$. m_2

Where does R-K come from?

slope of secant line is almost an average of m_1, m_2, and m_3

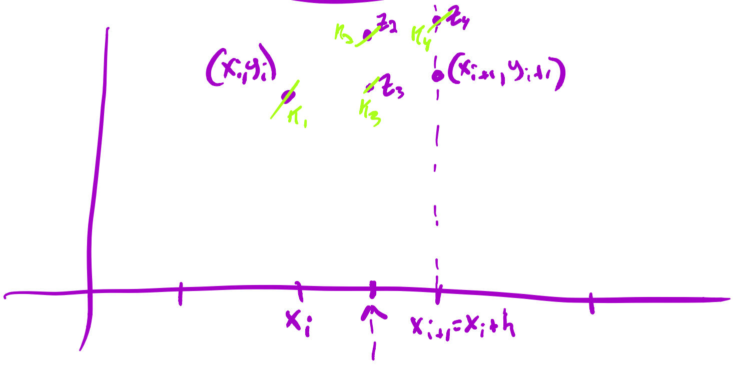
$$\boxed{\text{slope of secant} = \frac{m_1 + 4m_2 + m_3}{6}}$$

Runge-Kutta



$$\boxed{\text{slope} = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}}$$

Runge-Kutta



$x_i + 2h$

Initial Value Prob.
$y' = f(x,y)$
$y(a) = b$, h
Find $y(c)$

R-K - steps to move from (x_i, y_i) to (x_{i+1}, y_{i+1})

1. Start with (x_i, y_i)
2. $x_{i+1} = x_i + h$
3. $K_1 = f(x_i, y_i)$
4. $z_2 = y_i + \frac{h}{2} \cdot K_1$
5. $K_2 = f(x_i + \frac{h}{2}, z_2)$
6. $z_3 = y_i + \frac{h}{2} K_2$
7. $K_3 = f(x_i + \frac{h}{2}, z_3)$
8. $z_4 = y_i + h \cdot K_3$
9. $K_4 = f(x_i + h, z_4)$
10. $y_{i+1} = y_i + \frac{K_1 + 2K_2 + 2K_3 + K_4}{6} \cdot h$

Euler's
$y_{i+1} = y_i + K \cdot h$

Now we have our next point (x_{i+1}, y_{i+1})

- Office Hours 3-16 -

Example 1: Consider the initial value problem y'+2y=x^3 e^(-2x), y(0)=1. Approximate the value of y(0.6) using a step size of 0.3.

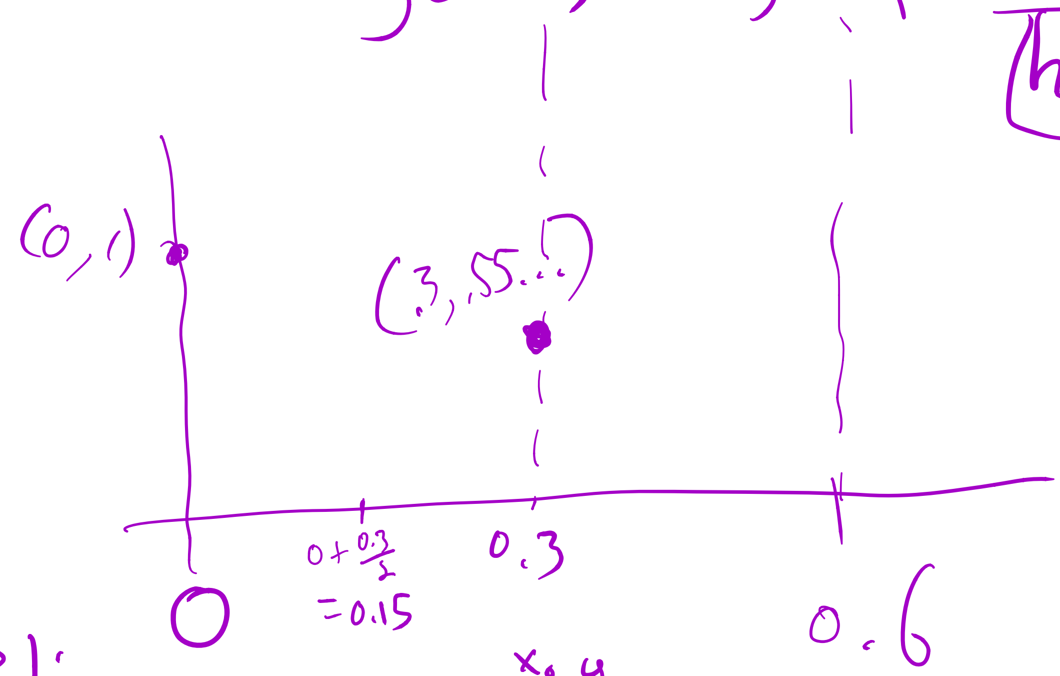
$$y' + 2y = x^3 e^{-2x}$$

$$-2y \qquad -2y$$

$$\boxed{y' = x^3 e^{-2x} - 2y} = f(x,y)$$

given $y(0) = 1$ $\quad (x_0, y_0) = (0,1)$

find $y(0.6)$ using step size 0.3 $\quad \boxed{h = 0.3}$



ROUND 1:

1. Start with $(\overset{x_0}{0}, \overset{y_0}{1})$

2. $x_1 = x_0 + h = 0 + 0.3 = 0.3$

3. $K_1 = f(0,1) = 0^3 e^{-2 \cdot 0} - 2 \cdot 1$

$K_1 = -2$

4. $Z_2 = y_0 + \frac{h}{2} \cdot K_1 = 1 + \frac{0.3}{2} \cdot (-2) = 0.7$

5. $K_2 = f\left(x_0 + \frac{h}{2}, Z_2\right) = f\left(0 + \frac{0.3}{2}, 0.7\right) = f(.15, 0.7)$

$K_2 = (.15)^3 e^{-2(.15)} - 2(0.7) = -1.3974997$

6. $Z_3 = y_0 + \frac{h}{2} K_2 = 1 + \frac{0.3}{2}(-1.3974997)$

$Z_3 = 0.790375$

7. $K_3 = f(.15, 0.790375) = (.15)^3 e^{-2(.15)} - 2(.790375)$

$K_3 = -1.578249817$

8. $Z_4 = y_0 + h K_3 = 1 + (0.3)(-1.578249817)$

$Z_4 = 0.526525054$

9. $K_4 = f(x_0 + h, Z_4) = f(0 + 0.3, 0.526525054\ldots)$

$= (.3)^3 e^{-2(.3)} - 2(0.526525054\ldots)$

$K_4 = -1.038232195\ldots$

$\boxed{-1.03823\ldots}$

10. $y_1 = y_0 + \frac{k_1 + 2k_2 + 2k_3 + k_4}{6} \cdot h$

$y_1 = 1 + \frac{-2 + 2(-1.3974997) + 2(-1.578249) +}{6}(.3)$

$y_1 = 0.5565134347$

Next point (x_{i+1}, y_{i+1})

$\boxed{(x_1, y_1) = (0.3, 0.5505134347)}$

Round 2:

1. Start with $(\overset{x_1}{0.3}, \overset{y_1}{0.5505134347})$
2. $X_2 = x_1 + h = 0.3 + 0.3 = 0.6$
3. $k_1 = f(x_1, y_1) = f(0.3, 0.5505134347)$

$k_1 = (0.3)^3 e^{-2(.3)} - 2(0.5505134347)$

$k_1 = -1.0862089523$

4. $z_2 = y_1 + \frac{h}{2} \cdot k_1 =$

$$z_2 = 0.5505134347 + \frac{0.3}{2}\left(-1.0862089\ldots\right)$$

$$\boxed{z_2 = 0.387582091415}$$

5. $$k_2 = f\left(x_1 + \frac{h}{2}, z_2\right)$$

$$= f\left(0.3 + \frac{0.3}{2}, 0.387582091415\right)$$

$$= f\left(0.45, 0.387582091415\right)$$

$$= (0.45)^3 e^{-2(0.45)} - 2(0.387582091415)$$

$$\boxed{k_2 = -0.738115522586}$$

6. $$z_3 = y_1 + \frac{h}{2} \cdot k_2$$

$$= 0.5505134347 + \frac{0.3}{2}\left(-0.738115522586\right)$$

$$\boxed{z_3 = 0.439796106312}$$

7. $$k_3 = f\left(x_1 + \frac{h}{2}, z_3\right)$$

$$= f\left(0.3 + \frac{0.3}{2}, 0.439796106312\right)$$

$$= f(.45, 0.439796106312)$$

$$k_3 = (.45)^3 e^{-2(.45)} - 2(.439796106\ldots)$$

$$\boxed{k_3 = -.8425435523}$$

8. $z_4 = y_1 + h k_3$

$$= .5505134347 + (0.3)(-.8425435523)$$

$$\boxed{z_4 = 0.29775036901}$$

9. $k_4 = f(x_1 + h, z_4)$

$$= f(0.3 + 0.3, 0.29775036901)$$

$$= f(.6, 0.29775036901)$$

$$= (.6)^3 e^{-2(.6)} - 2(.29775036901)$$

$$\boxed{k_4 = -0.5304427882}$$

10. $y_2 = y_1 + \dfrac{k_1 + 2k_2 + 2k_3 + k_4}{6} \cdot h$

5304427882

$y_2 = .5505134347 +$

$$\frac{-1.08620\ldots + 2(.7381155\ldots) + 2(-.84254\ldots)}{6}$$

$(.3)$

↑ times "h".

$$\boxed{y_2 = 0.31161494}$$

New point:

$(0.6, 0.31161494)$

$.5505134347 + \left[\left(-1.08626 + 2(.7381155) + 2(-.84254\ldots) + (-.53044278)\right) \div 6\right] * (.3)$

$y(0.6) \approx 0.31161494$ Final

Answer.

Example 1: Consider the initial value problem y'+2y=x^3 e^(-2x), y(0)=1. Approximate the value of y(0.6) using a step size of 0.3.

Example 1:	y'+2y=x^3 e^(-2x), y(0)=1		find y(0.6) using 2 steps					
i	**h**	**x_i**	**y_i**	**k1 = f(x_i,y_i)**	**k2=f(x_i+.5h,y_i+.5hk1)**	**k3=f(x_i+.5h, y_i+.5hk2)**	**k4=f(x+h,y+hk3)**	**Runge-Kutta y_(i+1) = y_i + h*(k1+2k2+2k3+k4)/6**
0	0.3	0	1	-2	-1.397499739	-1.578249817	-1.038232196	0.5505134347
1	0.3	0.3	0.5505134347	-1.086208955	-0.7381155225	-0.8425435523	-0.5304427882	0.31161494
2	0.3	0.6	**0.31161494**					

this is a good example of what I want your Numerica Method Calculator to produce (for the Rung-Kutta method)