General Problem

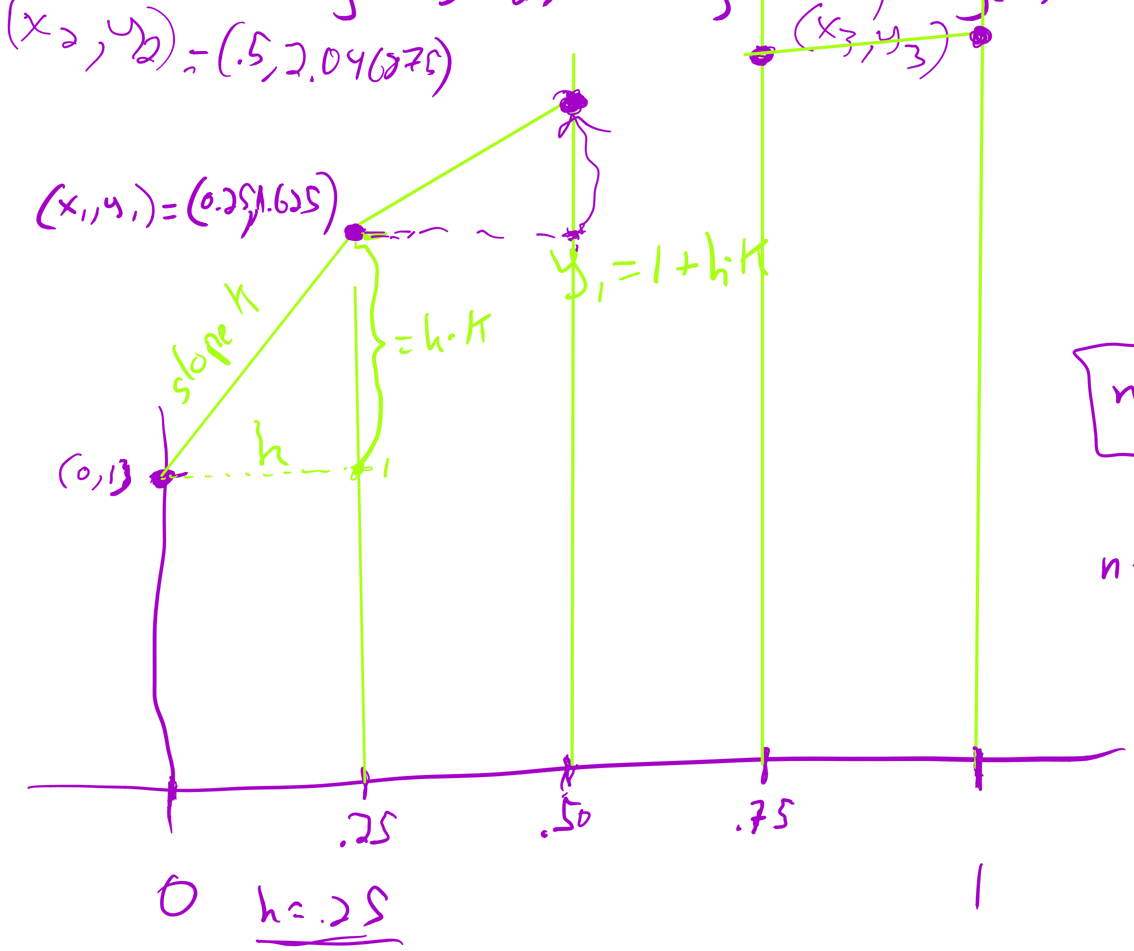
Start with a differential equation $y' = f(x,y)$ and a known value of the solution $y(a) = b$.

Goal: find another value $y(c)$, without actually solving the differential equation.

Example 1: Suppose y(x) is a solution to the initial value problem dy/dx=3-2x-0.5y, y(0)=1. Find the value of the function y at x=1 (find y(1)).
How do we do it?

$y' = 3 - 2x - 0.5y$ $y(0) = 1$, goal $y(1) = ?$

$y(1) \approx 2.379\ldots$



$$\boxed{n = \frac{c-a}{h}}$$

$$n = \frac{1-0}{.25} = \frac{1}{.25} = 4$$

Divide interval [0,1] into four equal pieces

step size $h = 0.25$

# of interval $n = 4$

sequence of points

$(x_0, y_0) = (0, 1)$

$(x_1, y_1) =$

$(x_2, y_2) =$

$(x_3, y_3) =$

$(x_4, y_4) =$

Note: can already state x-coords of each of these points:

$x_0 = 0$

$x_1 = .25$

$x_2 = .5$

$x_3 = .75$

$x_4 = 1$

Round 1: start $(x_0, y_0) = \underline{(0, 1)}$

slope at $(0,1)$

Substitute: $y' = 3 - 2x - 0.5y$

$y' = 3 - 2 \cdot 0 - 0.5 \cdot 1 = 2.5$

find (x_1, y_1)

$x_1 = 0.25$

$y_1 = y_0 + h \cdot K$

$y_1 = 1 + (.25)(2.5)$

$y_1 = 1.625$

$(x_1, y_1) = (0.25, 1.625)$

Round 2:

start $(x_1, y_1) = (0.25, 1.625)$

slope at $(0.25, 1.625)$

plug into $y' = 3 - 2x - 0.5y$

$y' = 3 - 2(.25) - 0.5(1.625)$

$y' = \underline{1.6875} = K$

$y_2 = y_1 + h \cdot K$

$y_2 = 1.625 + (.25)(1.6875)$

$y_2 = 2.046875$

$(x_2, y_2) = (.5, 2.046875)$

Round 3

start $(x_2, y_2) = \underline{(.5, 2.046875)}$

<u>slope</u>, substitute

$y' = 3 - 2(.5) - 0.5(2.046875)$

$y' = \boxed{0.9765625} = K.$

$y_3 = y_2 + h \cdot K$

$y_3 = y_2 + h \cdot k$

$y_3 = 2.046875 + (.25)(.978625)$

$y_3 = 2.291015625$

$(x_3, y_3) = (.75, 2.291015625)$

Round 4 start $(.75, 2.291015625)$

slope at

$y' = 3 - 2(.75) - 0.5(2.291015625)$

$y' = 0.354492187 5$

$y_4 = y_3 + h \cdot k$

$y_4 = 2.291015625 + (.25)(.3544921875)$

$y_4 = 2.37963867 2$

$$\boxed{(x_4, y_4) = (1, 2.379638672)}$$

$$\boxed{y(1) \approx 2.379638672}$$

Euler's Method. Given the differential equation y' = f(x,y) with initial condition y(a)=b, find an approximate value of the solution at x=c using step size *h*.

- Find a sequence of points (x_0,y_0), (x_1,y_1), (x_2,y_2)...(x_n,y_n)
  - NOTE: the number of steps *n* is related to the step size *h* by: h = (c-a)/n
- The first point (x_0,y_0) is given by the initial condition y(a)=b, so x_0=a and y_0=b
- x-coordinate: x_(i+1)=x_i + h
  You can write down the x-coordinates immediately, since h is the difference between successive x-values:
  $y_{i+1} = y_i + h \cdot K$
- y-coordinate: y_(i+1) = y_i + h*f(x_i,y_i)
  To approximate the value of y at x_(i+1), we use the slope of y at x_i

I recommend making a table:

n	x_n	y_n	f(x_n,y_n)	y_(n+1)