

Example (update): A n extra-hot cup of coffee at 180°F is carried outside in 22°F weather. After 5 minutes, the temperature of the coffee has dropped to 160°F. How long does it take the coffee to reach the perfect temperature of 130°F?

$T_m = 22^\circ$ temp. of environment. Guess: 10 min?

$$\begin{aligned} T(t) & \rightarrow T = 180 \text{ at } t=0, \text{ or } T(0) = 180 \\ & T = 160 \quad t=5 \quad T(5) = 160 \end{aligned}$$

STRATEGY ① plug in $T_m = 22$

$$T' = -K(T - 22)$$

solve this diff. equation
(treat K as constant)

$$T(t) = \frac{C}{e^{-Kt}}$$

(substitute $T(0) = 180$)

substitute $T(5) = 160$

Solve for K, C . what type of

$$T' = -K(T - 22) \quad \leftarrow \text{what type of} \quad \text{Diffy Q?}$$

$$T' = -KT + 22K$$

$$+KT \quad +KT$$

$$T' + KT = 22K \quad \text{first order linear nonhomogeneous.}$$

STEP 1: find single solution T_1 to the complementary eqn.

$$T' + KT = 0$$

$$-KT \quad -KT$$

$$\frac{T'}{T} = -\frac{KT}{T}$$

$$\left\{ \frac{T'}{T} dt = -K dt \right\}$$

$$\ln|T| = -Kt + C \quad \leftarrow \text{choose } C=0.$$

$$e^{\ln|T|} = e^{-Kt}$$

$$|T| = e^{-Kt}$$

$$T = \pm e^{-Kt}$$

$$T_1 = e^{-Kt}$$

choose :

$$\boxed{\frac{d}{dx} e^{-3x}} =$$

STEP 2: guess $T = u \cdot T_1$ *

$$\rightarrow T = u e^{-Kt}$$

$$T' = \underline{u} \cdot (-K) \cdot e^{-Kt} + e^{-Kt} \cdot u'$$

substitute:

$$T' + KT = 22K$$

$$-Ku e^{-kt} + \bar{e}^{-kt} \cdot u' + K \cdot u e^{-kt} = 22K$$

$$\bar{e}^{-kt} \cdot \bar{e}^{-kt} u' = 22K \cdot e^{-kt}$$

$$\int u' dt = 22K e^{-kt} dt$$

$$u = 22K \int e^{-kt} dt$$

$$u = 22K \frac{e^{-kt}}{-k} + C$$

$$u = 22e^{-kt} + C$$

$$T(t) = \underbrace{(22e^{-kt} + C) e^{-kt}}_{\boxed{T = 22 + C e^{-kt}}}$$

$$T(0) = 180$$

$$T(5) = 160$$

$$\text{Sub: } T = 180, t = 0$$

$$180 = 22 + C e^{-k \cdot 0}$$

$$180 = 22 + C \cdot e^0$$

$$\cancel{-22} \quad \cancel{-22}$$

$$\boxed{158 = C}$$

$$T = 160, t = 5$$

$$160 = 22 + C e^{-k \cdot 5}$$

$$160 = 22 + (58) e^{-5k}$$

$$\cancel{-22} \quad \cancel{-22}$$

$$\frac{138}{158} = \frac{158 e^{-5k}}{158}$$

$$\frac{138}{158} = e^{-5k}$$

$$\ln \left(\frac{138}{158} \right) = -5k$$

$$\frac{\ln\left(\frac{138}{158}\right)}{-5} = \frac{-5K}{-5}$$

$$\frac{\ln\left(\frac{138}{158}\right)}{-5} = K$$

$$K = 0.027068$$

$$T = 22 + 158 e^{-0.027068t}$$

find t when $T = 130$

$$130 = 22 + 158 e^{-0.027068t}$$

solve for t

$$t = 14.05571$$

Reorder S

130° at about

$$t = 14.1 \text{ min.}$$

New Chapter - Numerical Methods

Example 1: Suppose $y(x)$ is a solution to the initial value problem $dy/dx = 3 - 2x - 0.5y$, $y(0) = 1$. Find the value of the function y at $x=1$ (find $y(1)$).

How do we do it?

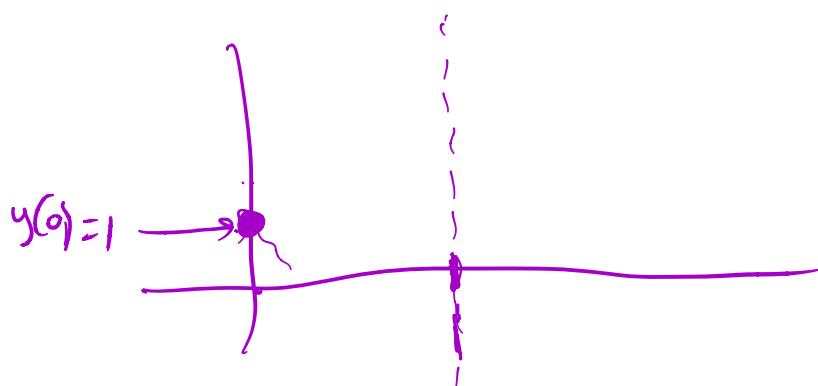
$$y' = 3 - 2x - 0.5y$$

$$y(0) = 1$$

Goal Find $y(1) =$

$$y(x) = ?$$

$$y(x) = ?$$



ONE STRATEGY: Solve the
differential equation (linear first order)

$$y = 14 - 4x - 13e^{-0.5x}$$

$$\text{Ans: } y(1) \approx 2.09$$

Ques: How can we get
this answer if we cannot solve the differential equation.

Numerical methods gives us a way.