Exact #2

M $\qquad F(x,y)=c$ $\qquad N$

$$\overbrace{3x^2+4-2xy}^{M}+\overbrace{(8-(x^2+6y^2))}^{N}y'=0$$

check $\qquad M_y = N_x$

$\frac{d}{dx}5=0$

$\frac{d}{dx}5^2=0$

$\frac{d}{dx}3\cdot5^2=0$

$\frac{d}{dx}3\cdot5^2\cdot x=75\cdot1$

$=3\cdot5^2\cdot1$

$$\frac{\partial}{\partial y}(3x^2+4-2xy) = -2x \checkmark$$

$$\frac{\partial}{\partial x}(8-x^2-6y^2) = -2x \checkmark$$

yes, this equation is exact.

(treat y as constant)

y's?

$$F(x,y)=\int 3x^2+4-2xy\,dx = x^3+4x-2y\left(\tfrac{1}{2}x^2\right)+\phi(y)$$

$$F(x,y)=x^3+4x-yx^2+\phi(y)$$

$\frac{\partial}{\partial y}3x^2 = 0$

$\frac{\partial}{\partial y}3x^2\cdot y = 3x^2\cdot1 = 3x^2$

treat x as constant

$$\frac{\partial}{\partial y}F(x,y)=\frac{\partial}{\partial y}\left(x^3+4x-yx^2+\phi(y)\right)$$

$$=0+0-x^2+\phi'(y)$$

$$=-x^2+\phi'(y)$$

Set $\frac{\partial}{\partial y}F = N$

$$-\cancel{x^2}+\phi'(y)=8-\cancel{x^2}-6y^2$$

$+x^2 \qquad\qquad +x^2$

$$\int\phi'(y)\,dy=\int 8-6y^2\,dy$$

$$\phi(y)=8y-2y^3+C$$

$\phi(y) = 8y - 2y^3 + C$

$$\boxed{F(x,y) = x^3 + 4x - yx^2 + 8y - 2y^3}$$

solution $\boxed{x^3 + 4x - 9x^2 + 8y - 2y^3 = C}$

Find the general solution for the following differential equation:
$16xy^2 + kx^2y + (8x^2 \cdot 2y - 5x^3)\, y' = 0$

For what value of k is this differential equation exact?

Using the value of k that you found above, solve the resulting differential equa

General Solution: $= c$

If you don't get this in 5 tries, you can get a hint.

Hint:

check $M_y = N_x$

x is constant
k is constant

$M_y = \frac{\partial}{\partial y}\left(16xy^2 + kx^2y\right) = 32xy + kx^2$

$N_x = \frac{\partial}{\partial x}\left(8x^2 \cdot 2y - 5x^3\right) = 32xy - 15x^2$

$16x^2y$

$$\boxed{K=-15}$$

$$16xy^2 - 15x^2y + (8x^2 - 2y - 5x^3)\cdot y' = 0$$

Solve

3. $ty' - 5y = t^4y^3$
4. $y' + 6y = -\left(2t + 12e^{-5t}\right),\ y(0) = -10$

Bernoulli:
$y' + p(t)y = F(t)y^n$

3. $\dfrac{ty' - 5y}{t} = \dfrac{t^4y^3}{t}$

$y' - \frac{5}{t}y = t^3y^3$ Bernoulli

① complementary: $y' - \frac{5}{t}y = 0$

$+\frac{5}{t}y \quad +\frac{5}{t}y$

$\dfrac{y'}{y} = \dfrac{\frac{5}{t}\cdot y}{y}$

$\int \frac{y'}{y}\,dt = \int \frac{5}{t}\,dt$

$y' = \dfrac{5y}{t}$

$$\ln|y| = 5\ln|t| + c$$

choose $c=0$ (any particular solution)

$$\ln|y| = 5\ln|t|$$

$$e^{\ln|y|} = e^{5\ln|t|}$$

$$|y| = e^{\ln|t^5|}$$

$$|y| = |t^5|$$

$$y = \pm t^5$$ choose "+" (single solution)

$$\boxed{y_1 = t^5}$$

$$e^{\ln|t^{\frac{1}{5}}|} \leftarrow e^{\frac{1}{5}\ln|t|}$$

Step 2 guess: $y = u\cdot y_1$

$$y = u\cdot t^5$$

$$y' = u\cdot 5t^4 + t^5\cdot u'$$

plug in

$$y' - \frac{5}{t}y = t^3y^3$$

$$u5t^4 + t^5\cdot u' - \frac{5}{t}(u\cdot t^5) = t^3(u\cdot t^5)^3$$

$$\cancel{5ut^4} + u't^5 - \cancel{5ut^4} = t^3u^3t^{5\cdot 3}$$

$$u't^5 = t^3u^3t^{15}$$

$$\frac{u'\cancel{t^5}}{\cancel{t^5}u^3} = \frac{t^{18}\cancel{u^3}}{t^5\cancel{u^3}}$$

$$\frac{u'}{u^3} = t^{18-5}$$

$$u'\cdot u^{-3} = t^{13}$$ -- "separate variables"

$$\int u^{-3}\underbrace{u'dt}_{du} = \int t^{13}\,dt$$

$$u^{-2} = t^{14} + C$$

$$\frac{\ \ }{-2} \qquad \frac{\ \ }{15}$$

$$u^{-2} = \frac{-2t^{15}}{15} - 2c$$

$+ C_{acceptable}$

$$\frac{1}{u^2} = \frac{-2t^{15}}{15} - 2c$$

$$u^2 = \frac{1}{\frac{-2t^{15}}{15} - 2c}$$

$$\left(u^{-2}\right)^{-\frac{1}{2}} = \left(\frac{-2t^{15}}{15} - 2c\right)^{-\frac{1}{2}}$$

$$\boxed{u = \left(\frac{-2t^{15}}{15} - 2c\right)^{-\frac{1}{2}}}$$

$$\frac{1}{u^2}$$

$$\sqrt{\frac{1}{u^2}}$$

$$\frac{1}{u} =$$

$$y = u y_1$$

$$\boxed{y = \left(\frac{-2t^{15}}{15} - 2c\right)^{-\frac{1}{2}} \cdot t^5}$$

$$y = \frac{1}{\left(\frac{-2t^{15}}{15} - 2c\right)^{1/2}} \cdot t^5$$

$$y = \frac{t^5}{\sqrt{\frac{-2t^{15}}{15} - 2c}} \cdot \frac{\sqrt{15}}{\sqrt{15}}$$

$$y = \frac{t^5\sqrt{15}}{\sqrt{\left(\frac{-2t^{15}}{15} - 2c\right)\cdot 15}}$$

$$= \frac{t^5\sqrt{15}}{\sqrt{-2t^{15} - 30c}}$$

$\sqrt{t^{30}} =$

$\left(t^{30}\right)^{1/2} = t^{15}$

$t^5 = \sqrt{t^{?}}$

$$\boxed{y = \frac{t^5\sqrt{15}}{\sqrt{-2t^{15} + \cancel{K}}}}$$

$$y = \pm\sqrt{\frac{7t^{10}}{c-t^{14}}}$$

$$y = \frac{\sqrt{t^{10}}\,\sqrt{15}}{\sqrt{-2t^{15} + K}}$$

How do we put something in a radical?

$$t^5 = \left(\left(t^5\right)^2\right)^{1/2}$$

$$y = \frac{\sqrt{15t^{10}}}{\sqrt{-2t^{15} + K}}$$

$$t^5 = \left(t^{10}\right)^{\frac{1}{2}} = \sqrt{t^{10}}$$

$$\boxed{y = \sqrt{\frac{15t^{10}}{-2t^{15} + K}}}$$

??

$$y = \pm\left(\sqrt{\frac{7-c}{t^{14}}}\right)t^5$$

$$= \sqrt{\frac{7-c}{t^{14}}}\sqrt{t^{10}}$$

$$=\sqrt{\frac{7-c}{\cancel{t^{14}}\;t^{4}}\cdot\cancel{t^{10}}}$$

$$y=\boxed{\pm\sqrt{\frac{7-c}{t^{4}}}}$$

~~Tehdi~~

$$\frac{u^{-2}}{-2}=\frac{t^{\cancel{15}\,14}}{\cancel{15}\,14}+c$$

u^{-2}

$\left(\frac{1}{u^2}\right)^{-\frac{1}{2}}=\sim$

$\sqrt{\frac{u^2}{1}}$

$u=\pm$

$$u^{-2}=-2\frac{t^{14}}{14}-2c$$

$$\left(u^{-2}\right)^{-\frac{1}{2}}=\left(-\frac{t^{14}}{7}-2c\right)^{-\frac{1}{2}}$$

$$u=\pm\left(-\frac{t^{14}}{7}-2c\right)^{-\frac{1}{2}}$$

$$y=u\cdot y_1$$

$$y = \pm\left(-\frac{t^{14}}{7} - 2c\right)^{-\frac{1}{2}} \cdot t^5$$

$$y = \pm \frac{1}{\sqrt{-\frac{t^{14}}{7} - 2c}} \cdot t^5$$

$$y = \pm \frac{\sqrt{t^{10}}}{\sqrt{-\frac{t^{14}}{7} - 2c}}$$

$$y = \pm\sqrt{\frac{t^{10}}{\left(-\frac{t^{14}}{7} - 2c\right)} \cdot \frac{7}{7}}$$

$$= \pm\sqrt{\frac{7t^{10}}{-t^{14} \underbrace{-14c}_{C_1}}}$$

$$\boxed{y = \pm\sqrt{\frac{7t^{10}}{-t^{14} + K}}}$$

$$\boxed{y = \pm\sqrt{\frac{7t^{10}}{c - t^{14}}}}$$