Separate the following differential equation and integrate to find the general solution:
$y^2e^{-x}y' = 3x$

Then give the particular solution that satisfies the initial condition $y(0) = 9.8$ and state the interval on which this solution is valid.

General Solution (explicitly): $y(x) =$

Particular Solution (explicitly): $y(x) =$

Interval of Validity:

Hint:

$$\frac{y^2 \cancel{e^{-x}} y'}{\cancel{e^{-x}}} = \frac{3x}{e^{-x}}$$

$$\int y^2 y' dx = \int 3xe^{x} dx$$

$$\frac{y^3}{3} = 3\int xe^{x} dx$$

$\int u\,dv = uv - \int v\,du$

$u = x \qquad dv = e^{x}dx$

$du = dx \qquad v = e^{x}$

$\int xe^{x}dx = xe^{x} - \int e^{x} dx$

$= xe^{x} - e^{x} + c$

$$\frac{y^3}{3} = 3(xe^{x} - e^{x} + c)$$

solve for y:

$$(y^3)^{\frac{1}{3}} = (9(xe^{x} - e^{x} + c))^{\frac{1}{3}}$$

$$\boxed{y = (9xe^{x} - 9e^{x} + 9c)^{\frac{1}{3}}}$$

General soln. explicit.

$y(0) = 9.8$

$\int e^{3x} dx$

u-substitution

$u = 3x$

$du = 3dx$

$\frac{du}{3} = dx$

$\int \frac{e^{u} du}{3}$

$\frac{1}{3}\int e^{u} du$

$\frac{1}{3}e^{u} + c$

$\boxed{\frac{1}{3}e^{3x} + c}$ *

Guess

e^{3x}

sub, solve for c:

$$9.8 = \left(\cancel{9\cdot 0e^{0}} - 9e^{0} + 9c\right)^{\frac{1}{3}}$$

$$(9.8)^3 = \left((-9+9c)^{\frac{1}{3}}\right)^3$$

$$9.8^3 = -9 + 9c$$

$$+9 \qquad +9$$

$$\frac{9.8^3 + 9}{9} = \frac{9c}{9}$$

calculator

$$c = \frac{9.8^3+9}{9} = \underline{105.576889}$$

Interval of validity: all real x

$(-\text{inf}, \text{inf})$

Specific solution:

$$y = \left(9xe^{x} - 9e^{x} + 9\cdot(105.576889)\right)^{\frac{1}{3}}$$

what is the domain of y?

$$y = \sqrt[3]{9xe^{x} - 9e^{x} + 950.192}$$

Domain is $\boxed{(-\infty, \infty)}$

check-test $\frac{d}{dx}$

$$\frac{d}{dx} e^{3x} = 3e^{3x}$$

Guess:

$$\boxed{\frac{1}{3}e^{3x} + c}$$

check:

$$\frac{1}{\cancel{3}}\cancel{3}e^{3x} = e^{3x}$$

Separate the following differential equation and integrate to find the general solution:

$y' = \frac{x^5}{y^4}$

General Solution (implicitly):

If you don't get this in 5 tries, you can get a hint.

Hint:

$y^4 \cdot y' = \frac{x^5}{y^4} \cdot y^4$

$\int y^4 y' dx = \int x^5 dx$

$\boxed{\frac{1}{5}y^5 = \frac{1}{6}x^6 + C}$ *

$$7y' = \frac{2x^1}{y^6 + x^2 y^6}$$

Then give the particular solution that satisfies the initial condition $y(0) = 7.4$ and stat
this solution is valid.

General Solution (explicitly): $y(x) =$

Particular Solution (explicitly): $y(x) =$

Interval of Validity:

If you don't get this in 5 tries, you can get a hint.

Hint:

$(y^6 + x^2 y^6)\, 7y' = \frac{2x}{\cancel{y^6 + x^2 y^6}} \cancel{(y^6 + x^2 y^6)}$

$(y^6 + x^2 y^6) \cdot 7y' = 2x$

$\frac{y^6(\cancel{1 + x^2}) \cdot 7y'}{\cancel{1+x^2}} = \frac{2x}{1+x^2}$

$\int 7y^6 y' dx = \int \frac{2x}{1+x^2} dx$

$u = 1 + x^2$

u-sub. or * inverse trig integrals

ex:

$du = 2x\,dx$

$\frac{7y^7}{7} =$

$\int \frac{1}{u}\,du = \ln|u| + c$

$= \ln|1+x^2| + c$

$\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + c$

$$\boxed{\left(y^7\right)^{\frac{1}{7}} = \left(\ln|1+x^2| + c\right)^{\frac{1}{7}}}$$ Implicit.

Solve for y

$$\boxed{y = \left(\ln|1+x^2| + c\right)^{1/7}}$$ explicit general solution.

Separate the following differential equation and integrate to find the general solution:
$y' = \cos^2(2x)\cos^2(4y)$

General Solution (implicitly):

If you don't get this in 5 tries, you can get a hint.

Submit Answers

$$y' = \cos^2(2x)\cos^2(4y)$$

$$\frac{y'}{(\cos(4y))^2} = \frac{(\cos(2x))^2\,\cancel{(\cos(4y))^2}}{\cancel{(\cos(4y))^2}}$$

$$\int \frac{y'}{(\cos(4y))^2}\,dx = \int (\cos(2x))^2\,dx$$

what is $\int \cos^2 x$? trig integral

Umbrella @9