Intervals of Convergence

Goal: y-

Separable Q4

x: $y' = (-2-2x)y^2$, $y(0) = \frac{-1}{63}$

NOTE: Separable

?

see below for finding general solution.

solution:

$y = \frac{1}{x^2+2x+C}$ || general solution

Find particular sol'n satisfying $y(0) = \frac{-1}{63}$, on what interval is it valid?

Sub $x=0$, $y=-\frac{1}{63}$ into general sol'n

$$-\frac{1}{63} = \frac{1}{0^2+2\cdot 0+C}$$

$$-\frac{1}{63} = \frac{1}{C}$$

solve for c

$$C = -63$$

$$\boxed{y = \frac{1}{x^2+2x-63}}$$ particular solution.

Domain of y: all numbers x except when denominator is 0.

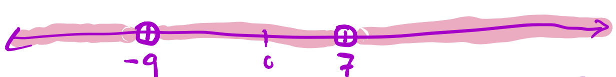
$$x^2+2x-63=0$$

$$(x+9)(x-7)=0$$

$x+9=0$ $\qquad$ $x-7=0$

$x=-9$ $\qquad$ $x=7$

Domain is all real numbers x except $x \neq -9$, $x \neq 7$



Interval of validity / Interval of convergence.

initial condition $y(0) = -\frac{1}{63}$

x-coordinate is $x=0$ $\longrightarrow$ x is in this intervals

Interval of validity is $\boxed{(-9, 7)}$

~~$y(\infty)=7$~~

if initial cond was $y(10) = $ ——

$x = 10$

(7 int)

Find general solution: $\frac{y'}{y^2} = \frac{(-2-2x)y^2}{y^2}$,

$$\int \frac{1}{y^2} \cdot y' \, dx = \int -2-2x \, dx$$

$$\int y^{-2} y' \, dx = -2x - x^2 + C$$

$$\frac{y^{-1}}{-1} = -2x - x^2 + C$$

$$\frac{-1}{y} = \frac{-2x - x^2 + C}{1}$$

solve for y:

$$(-1)\frac{y}{-1} = \frac{1}{-2x-x^2+C}(-1)$$

$$y = \frac{-1 \,(-1)}{-2x - \cancel{x}^2 + C \,(-1)}$$

$$\boxed{y = \frac{1}{2x + x^2 + C}}$$ general solution

Nonhomogeneous Linear

Find the general solution for the following differential equation:
$ty' + y = 3t\cos(7t)$

$y(t) =$

If you don't get this in 5 tries, you can get a hint.

Hint:

$$\frac{ty' + y}{t} = \frac{3t\cos(7t)}{t}$$

$$y' + p(t)y = f(t)$$

$y' + \frac{1}{t} \cdot y = 3\cos(7t)$ F.O.L first order linear nonhomogeneous

STEP 1 find a single solution $y_1(x)$ to the complementary equation:

$$y' + \frac{1}{t} y = 0$$

$$-\frac{1}{t}y \quad -\frac{1}{t}y$$

$$\frac{y'}{y} = \frac{-\frac{1}{t}y}{y}$$

$$\int \frac{y'}{y}\,dt = \int -\frac{1}{t}\,dt$$

$$\ln|y| = -\ln|t| + K$$

$$e^{\ln|y|} = e^{-\ln|t| + K}$$

$$|y| = e^K e^{-\ln|t|}$$

$$|y| = e^K e^{\ln|t^{-1}|}$$

$$|y| = e^K |t^{-1}|$$

$\frac{1}{t} = t^{-1}$

$$y = \pm e^k \left|\frac{1}{t}\right|$$

$y = \left|\frac{1}{t}\right|$ assume $t > 0$,

$$y = \frac{1}{t}$$

$\boxed{y_1 = \frac{1}{t}}$ — a soln to complementary.

STEP 2: $\boxed{\text{Guess } y = u \cdot y_1}$

$$y = u \cdot \frac{1}{t}$$

$$y' = u \cdot (-t^{-2}) + \frac{1}{t} \cdot u'$$

STEP 3: substitute into $y' + \frac{1}{t} y = 3\cos(7t)$

$$\cancel{u(-t^{-2})} + \frac{1}{t} \cdot u' + \cancel{\frac{1}{t}\left(u \cdot \frac{1}{t}\right)} = 3\cos(7t)$$

$$\frac{1}{t}\cdot u' = 3\cos(7t)$$

separate variables and integrate:

$$\cancel{t}\cdot\frac{1}{\cancel{t}}\cdot u' = 3\cos(7t)\cdot t$$

$$\int u'\,dt = \int 3t\cos(7t)\,dt$$

$$u = 3\int t\cos(7t)\,dt$$

integration by parts

$$\int t\cos(7t)\,dt =$$ Integration by Parts

$u = t \qquad dv = \cos(7t)\,dt$

$du = dt \qquad v = \int \cos(7t)\,dt$

u-substitution: $u = 7t$

$du = 7\,dt$

$\frac{1}{7}du = dt$

$\int \cos(u)\frac{1}{7}du$

$\sin(u)\cdot\frac{1}{7} + C$

$\sin(7t)\cdot\frac{1}{7} + C$

$$v = \frac{1}{7}\sin(7t)$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$ Int. by Parts

$$= t\left(\frac{1}{7}\sin(7t)\right) - \int \frac{1}{7}\sin(7t)\,dt$$

$$= \frac{t}{7}\sin(7t) - \frac{1}{7}\int \sin(7t)\,dt$$

$$= \frac{t}{7}\sin(7t) - \frac{1}{7}\left(-\frac{1}{7}\cos(7t) + C\right)$$

$$\boxed{= \frac{t}{7}\sin(7t) + \frac{1}{49}\cos(7t) + C_1}$$

$$u = 3\left(\frac{t}{7}\sin 7t + \frac{1}{49}\cos(7t) + C\right)$$

$$\boxed{u = \frac{3t}{7}\sin(7t) + \frac{3}{49}\cos(7t) + 3C}$$

$$y = u\cdot y_1$$

$$y = \left(\frac{3t}{7}\sin(7t) + \frac{3}{49}\cos(7t) + 3c\right)\cdot\frac{1}{t}$$

$$\boxed{y = \frac{3}{7}\sin(7t) + \frac{3}{49t}\cos(7t) + \frac{3c}{t}}$$

Final Answer

$$\frac{d}{dt}\cos t = -\sin t$$

$$\frac{d}{dt}\sin t = \cos t$$

check $\frac{d}{dt}\sin(7t) = 7\cos(7t)$

$$\frac{d}{dt}\boxed{\frac{1}{7}\sin(7t)} = \frac{1}{\cancel{7}}\cancel{7}\cdot\cos(7t) = \cos(7t)$$

$$e^{\ln \square} = \square$$

$$e^{5\ln x} = e^{\ln x^5} = x^5$$

$$e^{-\ln x} = e^{\ln x^{-1}} = x^{-1} = \frac{1}{x}$$