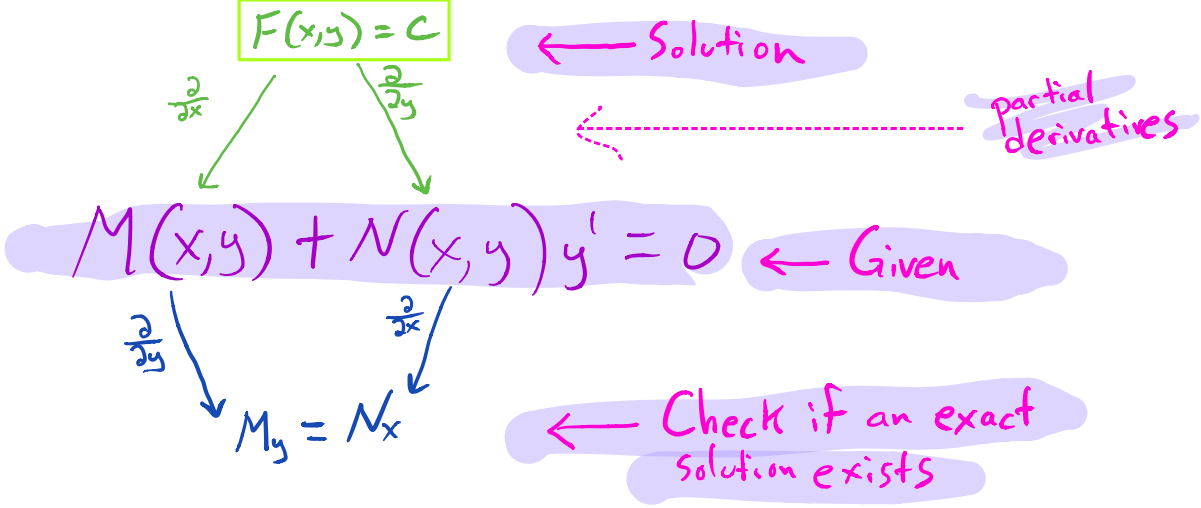


Exact Equations



SOLVING EXACT EQUATIONS:

Given a differential equation of the form: $M(x,y) + N(x,y)y' = 0$

1. Verify that the equation is **exact** by checking that $M_y = N_x$.
2. Integrate M with respect to x to obtain $F(x,y)$. Treat y as a constant. Don't forget to add a "constant" term $\varphi(y)$.
3. Take the partial derivative F_y and set it equal to N , solve for $\varphi'(y)$.
4. Integrate $\varphi'(y)$ to find $\varphi(y)$.
5. The general solution to the differential equation is given implicitly by: $F(x,y) = c$.

$$x^6 + x^2y^2 + y^5 = C$$

$$x^6 + x^2y^2 + \phi(y) = C$$

Example - Last Time

M N

← check

$$6x^5 + 2xy^2 + (2yx^2 + 5y^4) \frac{dy}{dx} = 0$$

$\frac{\partial}{\partial y}$ $\frac{\partial}{\partial x}$

$M_y = 0 + 4xy$ $N_x = 4yx + 0$

$M_y = N_x$ Yes, this equation is exact.

$$\frac{d}{dx} x^2 + \frac{7}{2}$$

$$2x + 0$$

Step 2: $\int M(x,y) dx$

$$= \int (6x^5 + 2xy^2) dx$$

treat y as constant.

$$= \frac{\cancel{6}x^6}{\cancel{6}} + 2y^2 \cdot \frac{x^2}{2} + C$$

$$= x^6 + y^2 x^2 + \phi(y)$$

$$F(x,y) = x^6 + y^2 x^2 + \phi(y)$$

↑ might include some y's

instead of

$C = \text{numerical constant}$

$\phi(y) = \text{function of } y$

Goal: find $\phi(y)$ to complete $F(x,y)$

Step 3: $\frac{\partial}{\partial y} F(x,y) = N(x,y)$

$$0 + \cancel{2yx^2} + \phi'(y) = \cancel{2yx^2} + 5y^4$$

$$\int \phi'(y) dy = \int 5y^4 dy$$

$$\phi(y) = \frac{5y^5}{5} = y^5$$

plug $\phi(y)$ back into $F(x,y)$

$$F(x,y) = x^6 + y^2 x^2 + y^5$$

$$\text{Solution: } F(x,y) = C$$

$$x^6 + x^2 y^2 + y^5 = C$$

Final answer

$$\left[\begin{array}{c|c} 7x & x \cdot 7 \\ \hline 7 & 7 \end{array} \right]$$

Intro- Population Modeling

Ex: A population of mice inhabit a field. Every month (month = 30 days), every pair of mice produces a single mouse baby. Every day, local owls eats 15 mice.

Ques: what happens to population over time?

$p(t)$ = population of mice at time t in months.

What is the initial population?

what if $p(0) = 600$ mice?

what if $p(0) = 1000$ mice?