Exact Equations

$$\boxed{F(x,y)=c}$$ ← Solution

$\frac{\partial}{\partial x}$ $\frac{\partial}{\partial y}$

← partial derivatives

$$M(x,y)+N(x,y)y'=0$$ ← Given

$\frac{\partial}{\partial y}$ $\frac{\partial}{\partial x}$

$$M_y=N_x$$ ← Check if an exact solution exists

SOLVING EXACT EQUATIONS:
Given a differential equation of the form: $M(x,y)+N(x,y)y'=0$
1. Verify that the equation is **exact** by checking that $M_y=N_x$.
2. Integrate M with respect to x to obtain $F(x,y)$. *Treat y as a constant. Don't forget to add a "constant" term* $\varphi(y)$.
3. Take the partial derivative F_y and set it equal to *N*, solve for $\varphi'(y)$.
4. Integrate $\varphi'(y)$ to find $\varphi(y)$
5. The general solution to the differential equation is given implicitly by: $F(x,y)=c$.

Example - Last Time

$$\boxed{x^6+x^2y^2+y^5=c}$$

$$\boxed{x^6+x^2y^2+\phi(y)=c}$$

Check

$$\underbrace{6x^5+2xy^2}_{M}+\underbrace{(2yx^2+5y^4)}_{N}\frac{dy}{dx}=0$$

$\frac{\partial}{\partial y}$ $\frac{\partial}{\partial x}$

$$M_y=0+4xy \qquad N_x=4yx+0$$

$$M_y=N_x$$ Yes, this equation is exact.

$$\frac{d}{dx}x^2+7$$

$$2x+0$$

Step 2: $\int M(x,y)\,dx$

$$=\int 6x^5+2xy^2\,dx$$

treat y as constant.

$$= \frac{\cancel{6}x^6}{\cancel{6}} + \cancel{2}y^2 \cdot \frac{x^2}{\cancel{2}} + \boxed{C}$$

might include some y's instead of C = numerical constant

$\phi(y)$ = function of y

$$= x^6 + y^2x^2 + \phi(y)$$

$$\boxed{F(x,y) = x^6 + y^2x^2 + \phi(y)}$$

Goal: find $\phi(y)$ to complete $F(x,y)$

Step 3: $\frac{\partial}{\partial y} F(x,y) = N(x,y)$

$$0 + \cancel{2yx^2} + \phi'(y) = \cancel{2yx^2} + 5y^4$$

$$-2yx^2 \qquad -2yx^2$$

$$\int \phi'(y)\,dy = \int 5y^4\,dy$$

$\frac{7x}{7}$ | $\frac{x \cdot 7}{7}$

$$\phi(y) = \frac{\cancel{5}y^5}{\cancel{5}} = y^5$$

plug $\phi(y)$ back into $F(x,y)$

$$\boxed{F(x,y) = x^6 + y^2x^2 + y^5}$$

solution: $F(x,y) = C$

$$x^6 + x^2y^2 + y^5 = C$$

Final answer

Intro - Population Modeling

Ex: A population of mice inhabit a field. Every month (month = 30 days), every pair of mice produces a single mouse baby. Every day, local owls eats 15 mice.

Ques: what happens to population over time?

$p(t)$ = population of mice at time t in months.

What is the initial population?

what if $p(0) = 600$ mice?

what if $p(0) = 1000$ mice?