Intervals of Convergence

Goal: $y = $ ___

SOLVING EXACT EQUATIONS:
Given a differential equation of the form: $M(x,y) + N(x,y)y' = 0$
1. Verify that the equation is **exact** by checking that $M_y = N_x$.
2. Integrate M with respect to x to obtain $F(x,y)$. *Treat y as a constant. Don't forget to add a "constant" term* $\varphi(y)$.
3. Take the partial derivative F_y and set it equal to *N*, solve for $\varphi'(y)$.
4. Integrate $\varphi'(y)$ to find $\varphi(y)$
5. The general solution to the differential equation is given implicitly by: $F(x,y) = c$.

Ex: $y' = (-2-2x)y^2, \quad y(0) = \frac{-1}{63}$

NOTE: Separable

?

solution:

$y = \frac{1}{x^2+2x+C}$ || general solution

Find particular sol'n satisfying $y(0) = \frac{-1}{63}$,
on what interval is it valid?
sub $x=0$, $y = -\frac{1}{63}$ into general sol'n

$$-\frac{1}{63} = \frac{1}{0^2+2\cdot 0 + C}$$

$$-\frac{1}{63} = \frac{1}{C}$$

Solve for c

$$C = -63$$

$$\boxed{y = \frac{1}{x^2+2x-63}}$$ particular solution.

Domain of y: all numbers x except when denominator is 0.

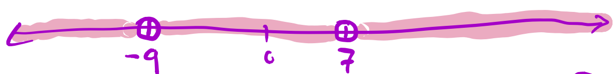
$$x^2+2x-63 = 0$$

$$(x+9)(x-7) = 0$$

$x+9=0$ $\qquad$ $x-7=0$

$x=-9$ $\qquad$ $x=7$

Domain is all real numbers x except $x \neq -9$, $x \neq 7$



Interval of validity / Interval of convergence.

initial condition $y(0) = -\frac{1}{63}$

x-coordinate is $x=0$ → x is in this intervals

Interval of validity is $\boxed{(-9, 7)}$

~~$y(\infty) = 7$~~

if initial cond was $y(10) = $ ___

$x = 10$

∞

interval: $(7, \text{inf})$

Exact Equations

Ex Solve $6x^5 + 2xy^2 + (2yx^2 + 5y^4)\frac{dy}{dx} = 0$

Verify that this: $\boxed{x^6 + x^2y^2 + y^5 = C}$ is an implicit solution.

To verify, take $\frac{d}{dx}$ of both sides:

$$\frac{d}{dx}(x^6 + x^2y^2 + y^5) = \frac{d}{dx} C$$

Implicit differentiation (i.e. y is a function of x)

$$6x^5 + x^2 \cdot 2y \cdot y' + y^2 \cdot 2x + 5y^4y' = 0$$

$$6x^5 + 2xy^2 + 2yx^2y' + 5y^4y' = 0$$

$$6x^5 + 2xy^2 + (2yx^2 + 5y^4)y' = 0$$

✓ yes, this is an implicit solution

Product Rule

$$\frac{d}{dx}\left(\overset{f}{x^2} \cdot \overset{g}{\sin x}\right) = f \cdot g' + g \cdot f'$$

$$x^2 \cos x + \sin x \cdot 2x$$

we refer to ~~the~~ the left side of the solution as a function

$$F(x,y) = x^6 + x^2y^2 + y^5$$

(*) In general, our goal will be to find $F(x,y)$ (then the solution will be $F(x,y) = C$)

Need: partial derivatives In a partial derivative, all other variables are treated as constants.

$F(x,y)$ has two partial derivatives:

$$\frac{\partial}{\partial x} F(x,y) = \frac{\partial}{\partial x}(x^6 + x^2y^2 + y^5) = 6x^5 + 2xy^2 + 0 = 6x^5 + 2xy^2$$

$$\frac{\partial}{\partial y} F(x,y) = \frac{\partial}{\partial y}(x^6 + x^2y^2 + y^5) = 0 + 2x^2y + 5y^4 = 2x^2y + 5y^4$$

$$\frac{d}{dx} x^2 \cdot 5$$

$$\frac{d}{dx} 5x^2 = 10x$$

$5 \cdot 2x$

$\frac{d}{dx} 10 = 0$

$\frac{d}{dx} 10^2 = 0$

$\frac{\partial}{\partial x} y = 0$

$\frac{\partial}{\partial x} y^5 = 0$

Put pieces together

An exact equation can be written in form: $M(x,y) + N(x,y)y' = 0$

Goal: find $F(x,y)$ so that

$F_x(x,y) \leftrightarrow \frac{\partial}{\partial x} F(x,y) = M(x,y)$ and

$F_y(x,y) \leftrightarrow \frac{\partial}{\partial y} F(x,y) = N(x,y)$

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F, F_x, F_y

$F_{xy} = F_{yx}$

1. check does $M_y = N_x$? If so, its exact.

Final answer: $\boxed{F(x,y) = c}$

ex: $\boxed{x^6 + x^2y^2 + y^5 = c}$

To be continued...