STRATEGY to solve $y' + p(x)y = f(x)$:
STEP 1: Solve the complementary equation $y' + p(x)y = 0$, let $y_1(x)$ be a solution.
STEP 2: Look for a solution of the form $y = uy_1$. Substitute into the original equation.
STEP 3: Solve for u by isolating u' and integrating.
STEP 4: Substitute $y_1(x)$, $u(x)$ to find the final solution: $y = uy_1$

Example: a) Find the general solution $y' + (\cot x)y = x\csc x$
b) Solve the initial value problem $y' + (\cot x)y = x\csc x$, $y(\pi/2) = 1$

Review

$$\frac{d}{dx}\ln x = \frac{1}{x}, \quad x>0$$

$$\frac{d}{dx}\ln|x| = \frac{1}{x}, \quad x\neq 0$$

Implicit Diff.

$$\frac{d}{dx}x^3 = 3\cdot x^2$$

$$\boxed{\frac{d}{dx}y^3} = \boxed{3\cdot y^2\cdot y'}$$

$$y' = \frac{dy}{dx}$$

$$= \boxed{3y^2\frac{dy}{dx}}$$

if y is a function of x

Heads up

y is (always) our function,
x or t is our independent variable

$$\frac{d}{dt}t^3 = 3t^2$$

Defn the order of a differential equation is the highest derivative that appears.

$$y' + x^3y'' + 7\sin xy = x^3 \quad // \text{order } 2$$

$$xy' = 2^x \quad / \text{ First order, order} = 1$$

First Order Differential Eq's

1st order $\sin(y') + x(y)^3 = 7xe^x$

1st order $y' + x^3 \cdot y = 7xe^x$

Linear First Order Diff. Eq.

has form:

$$y' + p(x)y = f(x)$$

any function involving x

homogeneous if $f(x) = 0$

$$\boxed{y' + p(x)y = 0}$$

homogeneous linear first order Diffy Q.

nonhomogeneous if $f(x) \neq 0$

$$\boxed{y' + p(x)y = f(x)}$$

$f(x)$

$f(x) = x^2 \sin x$

$f(x) = e^x$

$p(x)$

$p(x) = x^2 + 2x + 1$

$p(x) = \sin x \; e^x$

...

Example — homogeneous — what is y = ?

$$\boxed{y' - x^2y = 0}$$

$+x^2y \qquad +x^2y$

$$\frac{y'}{y} = \frac{x^2y}{y}$$

$$\int \frac{y'}{y}dx = \int x^2 dx$$

separate variables

xs on one side

ys on other "

$$= \frac{1}{3}x^3 + K$$

$$\int \frac{1}{y} \cdot y' \, dx = \frac{1}{3}x^3 + K$$

$$\ln|y| = \frac{1}{3}x^3 + K$$

$$e^{\ln|y|} = e^{\frac{1}{3}x^3 + K}$$

$$|y| = e^{\frac{1}{3}x^3} \cdot e^K$$

$$|y| = e^K \cdot e^{\frac{1}{3}x^3}$$

$$y = e^K \cdot e^{\frac{1}{3}x^3}$$
or
$$y = -e^K e^{\frac{1}{3}x^3}$$

$$\frac{d}{dx} \ln|x| = \frac{1}{x}$$

$$\frac{d}{dx} \ln|y| = \frac{1}{y} \cdot y'$$

family of solutions | general solution

$$\boxed{y = C \cdot e^{\frac{1}{3}x^3}}$$

$*\ y = 5e^{\frac{1}{3}x^3}$ ← two specific solutions

$*\ y = -7.2e^{\frac{1}{3}x^3}$ ←

Can we find $C = ?$ No.

If ~~and~~ we were given initial conditions like $y(3) = 7$, we could plug in and find c.

Nonhomogeneous case

Quick Preview (in more depth next time)

$$y' + p(x)y = f(x) \qquad y = \_\_\_\_$$

STEP 1 First, find a single solution y_1 to complementary equation
$$y' + p(x)y = 0$$

STEP 2 Guess $y = u(x) \cdot y_1$

STEP 3 plug in the guess into the original (nonhomogeneous) equation $y' + p(x)y = f(x)$.

STEP 4 solve for u (first solve for u', integrate to find u)

STEPS: solution is $y = u \cdot y_1$ $\square$