

# Differential Equations

## Intro and Review

GROUP WORK:

Example 3: Is  $y = x \sin x$  a solution to the differential equation  $y \sin x + y' \cos x - 1 = \sin x \cos x$ ?

Example 4: Find the solution to the differential equation  $y' = x\sqrt{x^2 + 8}$  satisfying  $y(1) = 11$ .

Example 5: Is  $y = \frac{1}{2} + e^{-x}$  a solution to the initial value problem  $y' + 2xy = x$ ,  $y(1) = \frac{1}{2}$ ?

Example 6: Find all solutions to  $y' = xe^y$ .

Example 7: Find the solution to  $y'' = x^5 + \sqrt{x^2 + x^2}$  satisfying  $y'(1) = \frac{23}{50}$  and  $y(1) = 2$ .

## Big Picture:

I am thinking of a function

$y = \underline{\hspace{2cm}}$  ← find the formula involving  $x$

Goal: find the function.

Ex: I'm thinking of  $y = \underline{\hspace{2cm}}$

where:

$$\begin{aligned} y' &= x^3 \\ y(1) &= 2 \end{aligned}$$

Proposed answer I

$$y = 3x^2$$

check:  $y' = 6x$

f(x)  
y(x)

$$\text{II } y = \frac{1}{4}x^4$$
$$y' = 4 \cdot \frac{1}{4}x^3$$
$$y' = x^3 \checkmark$$

$$y(1) = \frac{1}{4} \cdot 1^4 = \frac{1}{4}$$

$$\text{III } y = \frac{1}{4}x^4 + 1.75 \text{ (d)}$$

$$y' = x^3 \checkmark + 0 = \underline{x^3}$$

$$y(1) = \frac{1}{4} \cdot 1^4 + 1.75$$
$$= \frac{1}{4} + 1.75 = 2 \checkmark$$

Ex: I'm thinking of ~~y = 1/4 x^4~~  
where:  
y' = x^3 ← the differential equation  
y(1) = 2 ← initial condition(s).

$$\int y' dx = \int x^3 dx$$

$$y = \frac{1}{4}x^4 + C$$

This is a family of functions because of the "+ C"

We want a single function

Need to know C.

plug in y(1) = 2

so x = 1, y = 2

$$\rightarrow 2 = \frac{1}{4} \cdot 1^4 + C$$

$$2 = \frac{1}{4} + C$$

$$-\frac{1}{4} \quad -\frac{1}{4}$$

$$1\frac{3}{4} \text{ or } \frac{7}{4} \text{ or } 1.75 = C$$

$$y = \frac{1}{4}x^4 + 1\frac{3}{4} \quad \underline{\underline{\text{ANS}}}$$

Defn a Differential Equation is an equation involving one or more derivatives of an unknown function.  
 $y, y', y'', \dots$