Differential Equations

Intro and Review

Big Picture:

I am thinking of a function

$$y = \underline{\hspace{3cm}} \quad \leftarrow \text{find the formula involving } x$$

Goal: find the function.

Ex: I'm thinking of $y = \cancel{\text{~~~~~}}$
where:

$$\cancel{y' = x^3} \quad \Big\|$$

$$y(1) = 2 \quad \Big\|$$

Proposed answer I

$$y = 3x^2$$

check: $y' = 6x$

$$\text{II} \quad y = \frac{1}{4}x^4$$

$$y' = 4 \cdot \frac{1}{4}x^3$$

$$y' = x^3 \checkmark$$

$f(x)$

$y(x)$

$$y(1) = \frac{1}{4} \cdot 1^4 = \frac{1}{4}$$

$$\text{III} \quad \boxed{y = \frac{1}{4}x^4 + 1.75} \; \textcircled{*}$$

$$y' = x^3 \checkmark + 0 = \underline{x^3}$$

$$y(1) = \frac{1}{4} \cdot 1^4 + 1.75$$

$$= \frac{1}{4} + 1.75 = 2 \checkmark$$

Ex: I'm thinking of $y = \cancel{\mathit{wwww}}$

where:

$y' = x^3$ ← the differential equation

$y(1) = 2$ ← initial condition(s).

$$\int y' dx = \int x^3 dx$$

$$* \quad y = \frac{1}{4}x^4 + C \quad // \text{ this is a family of functions } \text{because the } "+C"$$

We want a single Function

Need to Know C.

plug in $y(1) = 2$

so $x = 1, y = 2$

$$2 = \frac{1}{4} \cdot 1^4 + C$$

$$2 = \frac{1}{4} + C$$

$$-\frac{1}{4} \quad -\frac{1}{4}$$

$$\boxed{1\frac{3}{4} \text{ or } \frac{7}{4} \text{ or } 1.75 = C}$$

$$\boxed{y = \frac{1}{4}x^4 + 1\frac{3}{4}} \underline{\text{ANS}}$$

Def<u>n</u> a <u>Differential Equation</u> is
an equation involving y', y'', etc.
one or more derivatives of
an <u>unknown Function</u> y.