[latexpage]

Numerical methods provide a way to compute (approximate) values of solutions to differential equations, even when we cannot solve the equations exactly. The drawback is the large number of numerical calculations required to obtain a desired value and level of precision. In this project, you will use technology to implement the various numerical methods and use your technological solution to solve differential equations problems.

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MAT 2680 Project

(Parts I-IV due Tuesday, April 13th) Use technology to compute approximate solutions to the initial value problem below using Euler's Method, the Improved Euler's Method, and the Runge-Kutta Method.

Initial Value Problem. Given the differential equation $dy/dx=2x-y$ and initial condition $y(0.2)=1.7$, find the value of $y(0.8)$.

Part I: Use technology (see "Technology Options" below) to approximate the solution to the Initial Value Problem using Euler's Method, the Improved Euler's Method, and the Runge-Kutta Method, each with a step size $h=0.05$. See "Project Requirements" below for more details.

Part II. Find the exact solution $y(x)$ to the Initial Value Problem and use it to determine the exact value of $y(0.8)$. Round your answer to 8 decimal places.
Type or write your answer to this part and submit it with your project.

Part III. Compare the exact value of $y(0.8)$ obtained in Part II to the three numerical approximations obtained in Part I. How many correct decimal digits did each method produce? Which method was the most accurate?
Type or write your answer to this part and submit it with your project.

Part IV. Reflection. Write one or two paragraphs (minimum 300 words) responding to the following. Leave your response to this part as a comment on this post.

1. Describe your project and how it works.
2. Describe the process of building your numerical methods calculator. What kind of technology did you decide to use, and why? Did you encounter any unexpected challenges in completing this project?
3. Why do we need numerical methods in addition to the other methods studied in the class?
4. Why is this assignment included in the class (instead of just computing the various methods using a calculator)?
5. Please include a link to your project (if it is online), or clearly state that you will be sending me the files by email (and don't forget to do it!).

Technology Options

You can choose your technology tool for this job – use any one of the following:

1. a spreadsheet (Excel, Google Sheets, or other spreadsheet)
    - if you choose to create a spreadsheet, you should have columns for $x, y, f(x,y)$, and so on, and each stage should appear in its own row (See Project Requirements below for more details)
    - HOW TO SUBMIT: If your project is a spreadsheet, either share it with me (if it is in Google Sheets or a similar cloud-based platform), or email the file to me as an attachment.
2. a programming language (Java, Perl, or other programming language)
    - if you choose to write code, your program should output the values of $x, y, f(x,y)$ and so on at each stage (See Project Requirements below for more details)
    - HOW TO SUBMIT: If your project is code, please submit it using an online coding site like ideone.com – once your code is working on the site, you can simply submit a link. If you are using a programming language not supported by ideone.com, you can email the source code to me.
3. mathematical software (MatLab, Maple, Mathematica, or other mathematical

software)
    - if you choose to use mathematical software, your program should output the values of $x, y, f(x,y)$ and so on at each stage (See Project Requirements below for more details)
    - HOW TO SUBMIT: If your project uses mathematical software, either share it with me (if it is in MatLab Online or a similar cloud-based platform), or email the file to me as an attachment.

Project Requirements

- Your solution must be able to carry out Euler's Method, Improved Euler's Method, and Runge-Kutta (you may implement these as three separate spreadsheets or programs if you wish).
- Your solution should display all the points (x,y) found along the way, not just the final point.
- Your solution should also display other values found while carrying out each method:
    1. Euler's Method: display the slope $f(x,y)$ at each stage
    2. Improved Euler's: display the values of $k1, k2$ at each stage
    3. Runge-Kutta: display the values of $k1, k2, k3, k4$ at each stage
    4. You can display other values as well, if you wish (for example, the intermediate y-value in the Improved Euler method that we refer to as z).
- Your solution may NOT use any built-in version of these methods (for example, most mathematical software contains a built-in command for Euler's Method – you can use this to check your work, but you need to create your own solution).

Test Your Project

Test your project. Data for the first few stages of calculation using each method appear below (this data was generated using a spreadsheet – Google Sheets)

Euler's Method

i	h	x_i	y_i	k = f(x_i,y_i)	y_(i+1)
0	0.05	0.2	1.7	-1.3	1.635
1	0.05	0.25	1.635	-1.135	1.57825
2	0.05	0.3	1.57825	-0.97825	1.5293375
3	0.05	0.35	1.5293375	-0.8293375	1.487870625

…

Improved Euler's Method

i	h	x_i	y_i	k1	z_(i+1)	k2	y_(i+1)
0	0.05	0.2	1.7	-1.3	1.635	-1.135	1.639125
1	0.05	0.25	1.639125	-1.139125	1.58216875	-0.98216875	1.586092656
2	0.05	0.3	1.586092656	-0.9860926563	1.536788023	-0.8367880234	1.540520639
3	0.05	0.35	1.540520639	-0.8405206393	1.498494607	-0.6984946073	1.502045258

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Runge-Kutta

i	h	x_i	y_i	k1 = f(x_i,y_i)	k2 = f(x_i+.5h,y_i+.5hk1)	k3 = f(x_i+.5h, y_i+.5hk2)	k4 = f(x+h,y+hk3)	y_(i+1) = y_i + h*(k1+2k2+2k3+k4)/6
0	0.05	0.2	1.7	-1.3	-1.2175	-1.2195625	-1.139021875	1.639057109
1	0.05	0.25	1.639057109	-1.139057109	-1.060580682	-1.062542592	-0.9859299798	1.585963496
2	0.05	0.3	1.585963496	-0.9859634957	-0.9113144083	-0.9131806355	-0.840304464	1.540336345
3	0.05	0.35	1.540336345	-0.8403363453	-0.7693279367	-0.7711031469	-0.701781188	1.501811514

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