Ex: Find the Inverse Laplace Transform:

a) $\frac{5}{(s-4)^2+25}$, s>4

b) $\frac{5}{s^2-8s+41}$, s>4

c) $\frac{1}{s-1}+\frac{4}{s+4}$, s>1

d) $\frac{5s}{s^2+3s-4}$, s>1

function	Laplace transform
1	$\frac{1}{s}$, $s>0$
e^{at}	$\frac{1}{s-a}$, $s>a$
t^n, n=positive integer	$\frac{n!}{s^{n+1}}$, $s>0$
$\sin at$	$\frac{a}{s^2+a^2}$, $s>0$
$\cos at$	$\frac{s}{s^2+a^2}$, $s>0$
$t\sin at$	$\frac{2as}{(s^2+a^2)^2}$, $s>0$
$t\cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$, $s>0$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$, $s>a$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$, $s>a$
$t^n e^{at}$, n=positive integer	$\frac{n!}{(s-a)^{n+1}}$, $s>a$
$af(t)+bg(t)$	$aF(s)+bG(s)$
$f'(t)$	$sF(s)-f(0)$
$f''(t)$	$s^2F(s)-sf(0)-f'(0)$

a) $\mathcal{L}^{-1}\left\{\frac{5}{(s-4)^2+25}\right\}$ s>4

a=4 b=5

$=\boxed{e^{4t}\sin 5t}$

b) $\frac{5}{s^2-8s+41}$, s>4

$\mathcal{L}^{-1}\left\{\frac{5}{s^2-8s+41}\right\}$

$=\mathcal{L}^{-1}\left\{\frac{5}{(s-4)^2+25}\right\}=\boxed{e^{4t}\sin 5t}$

claim:

$\frac{5}{s^2-8s+41}=\frac{5}{(s-4)^2+25}$

$=\frac{5}{s^2-8s+16+25}$

$=\frac{5}{s^2-8s+41}$

Rewriting the denominator $s^2-8s+41$ as $(s-4)^2+25$ is accomplished by completing the square

c) $\frac{1}{s-1}+\frac{4}{s+4}$, s>1

$\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}+\mathcal{L}^{-1}\left\{\frac{4}{s+4}\right\}$

a=1 a=−4

$=e^{t}+4\mathcal{L}^{-1}\left\{\frac{1}{s-(-4)}\right\}$

$=\boxed{e^{t}+4\cdot e^{-4t}}$

$\frac{1}{s-a}$ is there a value of a so that our formula looks like equals our function?

d) $\frac{5s}{s^2+3s-4}$, s>1

$\mathcal{L}^{-1}\left\{\frac{5s}{s^2+3s-4}\right\}$

$=\mathcal{L}^{-1}\left\{\frac{1}{s-1}+\frac{4}{s+4}\right\}$

$=\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}+\mathcal{L}^{-1}\left\{\frac{4}{s+4}\right\}$

$=\boxed{e^{t}+4e^{-4t}}$

Factor denom:

$s^2+3s-4=(s+4)(s-1)$

$\frac{5s}{(s+4)(s-1)}$

claim:

$\frac{5s}{(s+4)(s-1)}=\frac{1\cdot(s+4)}{(s-1)(s+4)}+\frac{4\cdot(s-1)}{(s+4)\cdot(s-1)}$

$=\frac{s+4}{(s-1)(s+4)}+\frac{4s-4}{(s+4)(s-1)}$

$=\frac{s+\cancel{4}+4s-\cancel{4}}{(s-1)(s+4)}=\frac{5s}{(s-1)(s+4)}$

the step where $\frac{5s}{s^2+3s-4}$ is rewritten as $\frac{1}{s-1}+\frac{4}{s+4}$ is done using the method of partial fractions.

Completing the square - allows you to rewrite as^2+bs+c without the "s" term: $a(s-m)^2+n$

ex: complete the square:

$$\boxed{s^2-12s+58}$$

divide by 2 and square it: 36

add and subtract this after the "s" term

$$\underbrace{s^2-12s+36}_{\text{factor}}\underbrace{-36+58}_{\text{combine}}$$

$$(s-6)(s-6)\quad+22$$

$$\boxed{(s-6)^2+22}$$

Ex partial fractions - can we write this as the sum of simpler fractions?

$$\frac{8x-44}{x^2-14x+45}$$

Factor denominator: $x^2-14x+45=(x-9)(x-5)$

Guess: $\dfrac{A}{x-9}+\dfrac{B}{x-5}$

$$\frac{8x-44}{(x-9)(x-5)}=\frac{A}{x-9}+\frac{B}{x-5}$$

Goal: find A, B

Multiply both sides by denominator on left

$$\cancel{(x-9)}\cancel{(x-5)}\cdot\frac{8x-44}{\cancel{(x-9)}\cancel{(x-5)}}=\left(\frac{A}{x-9}+\frac{B}{x-5}\right)\cdot(x-9)(x-5)$$

$$8x-44=A(x-5)+B(x-9)$$

$$8x-44=Ax-5A+Bx-9B$$

group like terms by "x"

$$8x-44=x(A+B)+(-5A-9B)$$

$$\boxed{\begin{aligned}8&=A+B\\-44&=-5A-9B\end{aligned}}$$

system 2 eq's 2 unknowns.

$$8=A+B$$
$$-B\qquad -B$$

$$\boxed{8-B=A}$$

$$-44=-5(8-B)-9B$$

$$-44=-40+\underbrace{5B-9B}$$

$$+40 \qquad +40$$

$$-4=-4B$$

$$\boxed{B=1}$$

$$A=8-B=8-1=7$$

$$\boxed{A=7}$$

thus

$$\boxed{\frac{8x-44}{(x-9)(x-5)}=\frac{7}{x-9}+\frac{1}{x-5}}$$