Recall

Definition a power series is a polynomial with infinitely many terms:

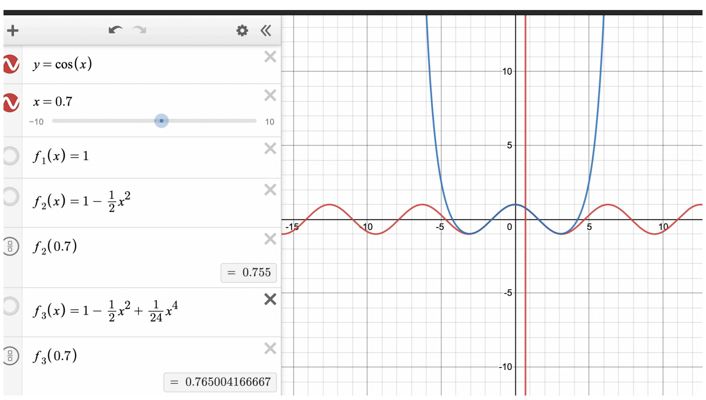
$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \dots$$

How do we answer questions like:

what is $\cos(0.7)$?

IDEA: basically any function can be approximated by polynomials, and, if use enough terms ($\to \infty$), we get a power series that is no longer an approximation - it's exactly equal to our function.

Consider the function $y = \cos(x)$. lets try to approximate it around $x=0$ with polynomials



Taylor / MacLaurin Series

Setup: if $f(x)$ is a function and $f(x)$ has derivatives of all orders at $x=0$

the the MacLourin Series of $f(x)$ is:

$$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots$$

if instead we consider a different point $x=c$, we get the Taylor Series of $f(x)$ at c

$$a_0 + a_1 (x-c) + a_2 (x-c)^2 + a_3 (x-c)^3 + a_4 (x-c)^4 + \dots$$

where the coefficients are given by:

$$a_n = \frac{f^{(n)}(c)}{n!}$$

Amazing fact: for most function, the Taylor Series/MacLourin series will exactly equal the function.

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$$

$$\cos(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \frac{1}{40320}x^8 + \dots$$

$\cos(0.7)$ $\boxed{} \overset{?}{=} \boxed{}$ $x=0.7$

close but not exactly equal

this is the "real" answer we want (infinitely many decimals)

the more terms we add, the closer we get to the "real" answer (the more digits are correct).

Ex 1: Given $y'' - xy = 0$, $y(0) = 3$, $y'(0) = 1$, estimate the value of $y(2)$ using the first eight terms of the power series.

STEP 1: guess

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \dots$$

what do we do with our guess?

find derivatives y', y'',

substitute into the original equation

try to find values of the coefficients

$a_0, a_1, a_2, a_3, \ldots$