Example 1: Suppose y(x) is a solution to the initial value problem
$y' = 3 - 2x - 0.5y,\ y(1) = 0.6$. Find an approximate value of y(2) using Euler's method with step size h=0.5.

i	h	x_i	y_i	k = f(x_i,y_i)	y_(i+1)
0	0.5	1.0	0.6	0.7	.95
1	0.5	1.5	0.95	-0.475	0.7125
2	0.5	2.0	**0.7125**		

y_(i+1) = y_i + k*h = 0.6 + 0.7*0.5=.95

y2=y1+kh=0.95 + (-0.475)(0.5)=0.7125

ANS: y(2)==0.7125

QUESTIONS: How close is this approximation to the correct answer?
How can we improve our approximation?

ACTUAL SOLUTION

NOTE: The solution to this initial value problem is:
y(x) = -4 x - 15.498 e^(-0.5 x) + 14
"CORRECT" ANS: ***y(2) = 0.298612***

IMPROVED EULER FORMULA: yn+1=yn+f(tn,yn)+f(tn+h, yn+hfn)2h
NOTE: This is a "two step" method -- first we calculate z_(i+1) = y_i+h f(x_i,y_i), then we use that value to plug in and calculate y_(i+1).

IMPROVED EULER METHOD
Given a point (x_i,y_i), how do we find the next point, (x_(i+1),y_(i+1))
Calculate:
Find x_(i+1) = x_i + h
Find k1=f(t_i, y_i)
Find z_(i+1) = y_i + h*k1
Find k2 = f(t_(i+1), z_(i+1))
Find y_(i+1)=y_i+h * [k1+k2]/2
Now we have (x_(i+1),y_(i+1))

Example 2: Suppose y(x) is a solution to the initial value problem dy/dx=3-2x-0.5y, y(1)=0.6. Find an approximate value of y(2) using the Improved Euler's method with step size h=0.5.
"CORRECT" ANS: y(2) = 0.298612

i	h	x_i	y_i	k1	z_(i+1)	k2	y_(i+1)
0	0.5	1	0.6	0.7	0.95	-0.475	0.65625
1	0.5	1.5	0.65625	-0.328125	0.4921875	-1.24609375	0.2626953125
2	0.5	2	**0.2626953125**				

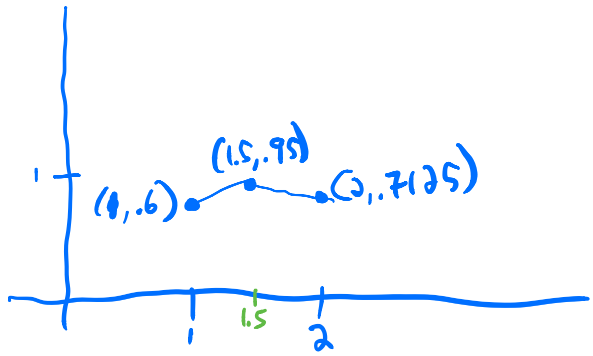
ROUND 1:
k1 = f(1,0.6)=3-2x-0.5y = 3-2(1)-0.5(0.6) =
z = y_i + k1*h=0.6+0.7*0.5 = 0.95 ← this is a temporary y-value
k2 = f(1.5,0.95)=3-2(1.5)-0.5(.95) = -0.475 ← slope at right side of interval
y_(i+1)=y_i + (k1+k2)/2 * h = 0.6 + (0.7 -0.475)/2 *(0.5) = 0.65625

ROUND 2:
k1 = f(1.5,0.65625) = 3-2(1.5)-0.5(.65625) = -0.328125
z = 1.5 + -0.328125 * (0.5) = 0.4921875
k2 = f(2,0.4921875) = 3-2(2)-0.5(0.4921875)=-1.24609375
y_(i+i)=0.65625 + (-0.328125+-1.24609375)/2 *(0.5)= 0.2626953125

ANS: according to Improved Euler's Method, **y(2) == 0.2626953125**

COMPARE:
Euler's: **y(2)==0.7125**
Improved Euler's: **y(2) == 0.2626953125**
Actual Value: ***y(2) = 0.298612***

Example 1: Suppose y(x) is a solution to the initial value problem
$y' = 3 - 2x - 0.5y,\ y(1) = 0.6.$ Find an approximate value of y(2) using Euler's method with step size h=0.5.



slope $f(x,y) = 3 - 2x - 0.5y$

initial point: $(a,b) = (1, 0.6)$

goal point $c = x = 2$ $\quad (2, ?)$

$h = 0.5$

$n = \frac{c-a}{h} = \frac{2-1}{0.5} = \frac{1}{0.5} = \boxed{2}$

i	h	x_i	y_i	$K = f(x_i, y_i)$	y_{i+1}
0	0.5	1	0.6	0.7	0.95
1	0.5	1.5	0.95	-0.475	0.7125
2	0.5	2	.7125		

$f(x,y) = 3 - 2x - 0.5y$

Scratch

$K = f(1, 0.6)$
$= 3 - 2\cdot 1 - 0.5(0.6)$
$= 0.7$

$y_{i+1} = y_i + K\cdot h$
$= 0.6 + 0.7(0.5) = 0.95$

$x_{i+1} = x_i + h$

$K = f(1.5, 0.95) = 3 - 2(1.5) - 0.5(.95)$
$= -0.475$

$y_{i+1} = y_i + Kh = .95 + (-0.475)(.5)$
$= 0.7125$

ANSWER: $y(2) \approx 0.7125$

How close is this to the actual answer $y(2)$?

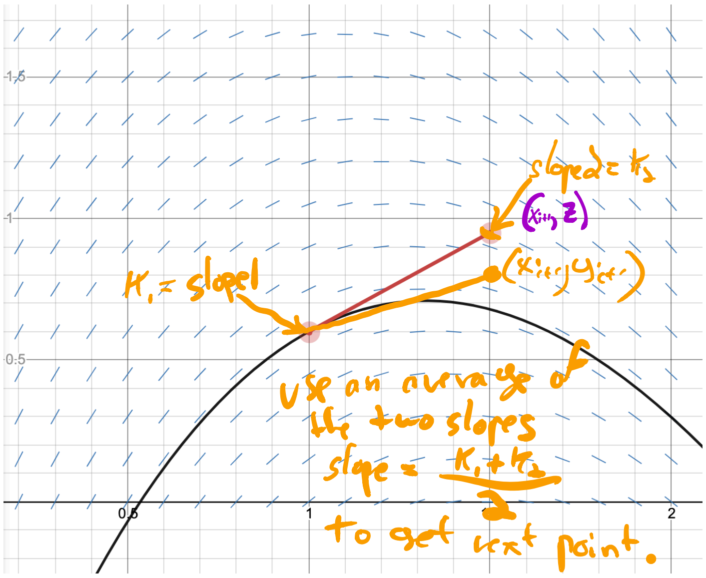
How do we get this?

Note: actual solution is:

$$y = -4x - 15.498e^{-0.5x} + 14$$

$$y(2) = -4(2) - 15.498e^{-0.5(2)} + 14$$

$$\boxed{y(2) = 0.298612}$$



Improved Euler's Method

set up is same:

$y' = f(x, y)$

initial point (a, b)

$h =$

goal point $x = c$

How do we find the next point (x_{i+1}, y_{i+1}) given (x_i, y_i)?

STEPS:

$x_{i+1} = x_i + h$

$K_1 = f(x_i, y_i)$

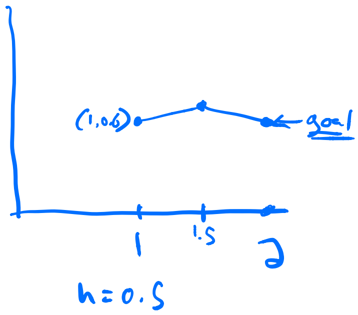
$z = y_i + K_1 \cdot h$

average of slopes.

$$K_2 = f(x_{i+1}, z)$$

$$y_{i+1} = y_i + \left(\frac{K_1+K_2}{2}\right)\cdot h$$

Example 2: Suppose y(x) is a solution to the initial value problem dy/dx=3-2x-0.5y, y(1)=0.6. Find an approximate value of y(2) using the Improved Euler's method with step size h=0.5.



i	h	x_i	y_i	$K_1=f(x_i,y_i)$	z	$K_2=f(x_{i+1},z)$	y_{i+1}
0	0.5	1	0.6	0.7	0.95	-0.475	0.65625
1	0.5	1.5	.65625	-.328125	0.4928175	-1.246...	0.2626953
2	0.5	2	$\boxed{0.2626953}$				

Scratch

$$K_1 = f(1, 0.6) = 3-2(1)-0.5(0.6) = 0.7$$

$$z = y_i + K_1\cdot h = 0.6+0.7(0.5) = 0.95$$

$$K_2 = f(1.5, 0.95) = 3-2(1.5)-0.5(0.95) = -0.475$$

$$y_{i+1} = y_i + \frac{K_1+K_2}{2}\cdot h = 0.6 + \left(\frac{0.7+(-.475)}{2}\right)\cdot(0.5)$$

$$= 0.65625$$

$= 0.65625$

Starting at $(1.5, 0.65625)$

$$K_1 = f(1.5, 0.65625) = 3 - 2(1.5) - 0.5(.65625)$$
$$= -0.328125$$

$$z = y_i + K_1 \cdot h = 0.65625 + (-.328125)(0.5)$$
$$= 0.4921875$$

$$K_2 = f(x_{i+1}, z) = f(2, 0.4921875)$$
$$= 3 - 2(2) - 0.5(.4921875)$$
$$= -1.24609375$$

$$y_{i+1} = y_i + \left(\frac{K_1 + K_2}{2}\right) \cdot h$$
$$= .65625 + \frac{-0.32815 + (-1.24609375)}{2}(.5)$$
$$= \boxed{0.262695 3}$$

ANS: $y(2) \approx 0.2626953$ Improved Euler

$y(2) \approx 0.7125$ Euler's

$\boxed{y(2) = 0.298612}$ Actual

Office Hours 10/12

Newton's Law of Cooling

$$T' = -k(T - T_m)$$

Modeling Cooling: Problem 5

(4 points)

Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/9_Introduction_to_Differential_Equations/9.2_Models_Involvin
b/9.2.10.pg

This set is **visible to students.**

When a hot object is placed in a water bath whose temperature is $25^\circ\mathrm{C}$, it cools from $100^\circ\mathrm{C}$ to $50^\circ\mathrm{C}$ in $180\mathrm{s}$. In another bath, the same cooling occurs in $160\mathrm{s}$. Find the temperature of the second bath.

The temperature of the second bath = ☐ $^\circ\mathrm{C}$

Solution:

Bath 1

$T_m = 25$

$T(0) = 100$

$T(180) = 50$

$T' = -k(T - 25)$

solve for T

plug in $T(0) = 100$

Bath II

$T_m = ?$

$T(0) = 100$

$T(160) = 50$

$k = -.00225$

$T' = -(-.00225)(T - T_m)$

solve again,

$T(180) = 50$

to find K, C.

plug in $T(0) = 100$

$T(160) = 50$

to get C, T_m.

$$\boxed{K = .00225}$$

Start:

$$T' = -K(T - 25)$$

$$\frac{T'}{-K(T-25)} = \frac{-K(T-25)}{-K(T-25)}$$

$$\int \frac{1}{-KT + 25K} \cdot T' \, dt = \int 1 \, dt$$

use u-substitution

$$\frac{\ln|-KT + 25K|}{-K} = t + C$$

$$\ln|-KT + 25K| = -Kt - KC$$

$$e^{\ln|-KT+25K|} = e^{-Kt - KC}$$

$$|-KT + 25K| = e^{-Kt} e^{-KC} \quad = C_1$$

$$-KT + 25K = \pm C_1 e^{-Kt}$$

$T =$

e^{-Kt} (180)

$50 = 100 + -75e^{-K180}$ formula may be off?

$-50 = -75e^{-K180}$

$$-kT + 25k = C_2 e$$

$$\frac{-kT}{-k} = \frac{C_2 e^{-kt} - 25k}{-k}$$

$$T = \frac{C_2}{-k} e^{-kt} + 25$$

$$\boxed{T = C_3 e^{-kt} + 25}$$

plug in $T(0) = 100$

$T(180) = 50$

to find C, K.

$$100 = C_3 e^{-k \cdot 0} - 25$$

$$\boxed{125 = C_3}$$

$$T = 125 e^{-kt} - 25$$

plug in $T(180) = 50$

$$50 = 125 e^{-k(180)} - 25$$

solve k:

Solve x,