SOLVING EXACT EQUATIONS:
Given a differential equation of the form: $M(x, y) + N(x, y)y' = 0$
1. Verify that the equation is **exact** by checking that $M_y = N_x$.
2. Integrate M with respect to x to obtain $F(x, y)$. *Treat y as a constant. Don't forget to add a "constant" term* $\phi(y)$.
3. Take the partial derivative F_y and set it equal to N, solve for $\phi'(y)$.
4. Integrate $\phi'(y)$ to find $\phi(y)$
5. The general solution to the differential equation is given implicitly by: $F(x, y) = c$.

yesterday: $F(x,y) = y\sin x + x^2e^y - y$

Example 3. Solve $(y\cos x + 2xe^y) + (\sin x + x^2e^y - 1)y' = 0$
ANS: $y\sin x + x^2e^y - y = c$

Example 4. Solve $(3xy + y^2) + (x^2 + xy)y' = 0$

$M = 3xy + y^2$

$N = x^2 + xy$

STEP 1 check if $M_y = N_x$

$M_y = 3x + 2y$

$N_x = 2x + y$

$M_y \neq N_x$

This equation is not exact ☑ DONE

SO this method will not work to find the solution.

$$\begin{tikzcd}
& F(x,y) \arrow[dl, "F_x"'] \arrow[dr, "F_y"] & \\
F_x = M(x,y) \arrow[dr] & & N(x,y) = F_y \arrow[dl] \\
& M_y = N_x &
\end{tikzcd}$$

$\leftarrow$ check for exactness

Exact: Problem 5

(4 points) local/set5-Exact/problem5.pg

This set is **visible to students.**

Find the general solution for the following differential equation:
$4xy^2 + kx^2y + (2x^2 \cdot 2y - 7x^3)y' = 0$

For what value of k is this differential equation exact? [ ]

Using the value of k that you found above, solve the resulting differential equation.
General Solution: [ ] $= c$

If you don't get this in 5 tries, you can get a hint.

Hint:

$M = 4xy^2 + kx^2y$

$N = 2x^2 \cdot 2y - 7x^3 = 4x^2y - 7x^3$

check $M_y = N_x$

$M_y = 4x \cdot 2y + kx^2 = 8xy + kx^2$

$N_x = 4y \cdot 2x - 7 \cdot 3x^2 = 8xy - 21x^2$

Its exact if $M_y = N_x$

$\underline{k = -21.}$

Our equation is:

$$4xy^2 - 21x^2y + (2x^2 2y - 7x^3)y' = 0$$

$$M = 4xy^2 - 21x^2y$$

$$N = 2x^2 \cdot 2y - 7x^3$$

(we know it's exact from above)

STEP 2: $\int 4xy^2 - 21x^2y \, dx$ treat y as constant.

"phi" = "Fee"

$$F(x,y) = 4y^2 \cdot \frac{x^2}{2} - 21y\frac{x^3}{3} + \phi(y)$$

$$\boxed{F(x,y) = 2x^2y^2 - 7x^3y + \phi(y)}$$

STEP 3: $F_y = \underline{2x^2 \cdot 2y} - 7x^3 + \phi'(y)$

Set $F_y = N$

$$4x^2y - 7x^3 + \phi'(y) = 4x^2y - 7x^3$$

$$\int \phi'(y) = \int 0$$

$$\phi(y) = C.$$

$$F(x,y) = 2x^2y^2 - 7x^3$$

general solution:

$$\boxed{2x^2y^2 - 7x^3 = C.}$$

Modeling

assume a month is 30 days.

Example 1: A population of field mice inhabit a rural area. Some owls live in the area. Let's track the population p(t) over time (t in months).

- every month, each pair of mice produces one new mouse.
- every night, the owls eat 15 mice.

How many mice are killed each month? $15 \cdot 30 = 450$ mice per month
How many mice are born each month? → depends on population.
Does the population go up, or down? → depends on the initial population. Is the amount added by reproduction more or less than the amount killed by predation?

Two scenarios: How many mice after 1 month if the starting population is:

A. $p(0) = 600$ mice
added by reproduction: 300
killed by owls: 450
total after 1 month: $600 + 300 - 450 = 450$

B. $p(0) = 1000$ mice
reproduction: 500
killed by owls: 450
total: $1000 + 500 - 450 = 1050$

Homogeneous - y over x: Problem 2

(4 points) local/set3-Separable/problem8.pg

This set is **visible to students.**

Use a substitute variable to replace y in order to make this DE separable, then separate and solve: Show Problem Sou

$y' = \frac{9x^2+4y^2}{2xy}$

General Solution (implicitly for y^2): $[y(x)]^2 =$

If you don't get this in 5 tries, you can get a hint

$$y' = \frac{9x^2+4y^2}{2xy}$$

Goal: $y' = f\left(\frac{y}{x}\right)$

$$y' = \frac{9x^2}{2xy} + \frac{4y^2}{2xy}$$

$$y' = \frac{9x}{2y} + \frac{2y}{x}$$

$$y' = \left(\frac{9}{2}\right)\left(\frac{x}{y}\right) + 2\cdot\frac{y}{x}$$

$$y' = \left(\frac{9}{2}\right)\left(\frac{y}{x}\right)^{-1} + 2\cdot\frac{y}{x}$$

STEP1: let $y_1 = x$

STEP2: Guess $y = u\cdot y_1$

$\boxed{y = ux}$

$\boxed{\frac{y}{x} = u}$

$y' = u\cdot 1 + u'\cdot x$

Substitute:

$u + u'x = \left(\frac{9}{2}\right)\cdot u^{-1} + 2u$

Separate variables

$-u$ $\quad -u$

$$u'x = \frac{9}{2}\frac{1}{u} + 2u - u$$

$$\frac{1}{x}\cdot u'x = \frac{9}{2}\cdot\frac{1}{u} + u \cdot\frac{1}{x}$$

$$u' = \left(\frac{9}{2}\frac{1}{u} + u\right)\cdot\frac{1}{x}$$

divide by

$$\int \frac{u'}{\frac{9}{2}\frac{1}{u} + u}\,dx = \int \frac{1}{x}\,dx$$

$$\int \frac{1}{\frac{9}{2u} + u}\,u'dx = \ln|x| + C$$

simplify - multiply top and bottom by $2u$

$$\int \frac{1}{\frac{9}{2u} + u}\cdot\frac{2u}{2u}\,u'dx =$$

$$\int \frac{2u}{9 + 2u^2}\frac{du}{dx}\,dx =$$

Use substitution to integrate

let $\boxed{w = 9+2u^2}$

$\frac{dw}{4} = \frac{4u\,du}{4}$

$\boxed{\frac{1}{4}dw = u\,du}$

sub:

$$\int \frac{2}{w} \cdot \frac{1}{4}dw$$

$$\frac{1}{2}\int \frac{1}{w}dw$$

$$\frac{1}{2}\ln|w|$$

$$\frac{1}{2}\ln|9+2u^2| = \ln|x| + C$$

solve for u:

$$\ln|9+2u^2| = 2\ln|x| + 2C$$

$$e^{\ln|9+2u^2|} = e^{2\ln|x| + 2C}$$

$$|9+2u^2| = e^{2\ln|x|+2C}$$

$$|9+2u^2| = e^{2\ln|x|} e^{2C}$$

$$|9+2u^2| = e^{\ln|x^2|} \cdot e^{2C}$$

$$|9+2u^2| = |x^2| \cdot e^{2C}$$

nonnegative. x^2 is nonnegative, so $|x^2| = x^2$

$$9+2u^2 = x^2 \cdot e^{2C}$$

-9 $\quad$ -9

$$2u^2 = x^2 \cdot e^{2C} - 9$$

$$u^2 = \frac{1}{2}x^2e^{2c} - \frac{9}{2}$$

$$u = \pm\sqrt{\frac{1}{2}x^2e^{2c} - 9/2}$$

$$y = ux$$

$$y = \left(\pm\sqrt{\frac{1}{2}x^2e^{2c} - \frac{9}{2}}\right)\cdot x$$

the problem asks for y^2, so lets square both sides:

$$y^2 = \left[\left(\pm\sqrt{\frac{1}{2}x^2e^{2c} - \frac{9}{2}}\right)\cdot x\right]^2$$

$$y^2 = \left(\pm\sqrt{\frac{1}{2}x^2e^{2c} - \frac{9}{2}}\right)^2\cdot x^2$$

$$y^2 = \left(\frac{1}{2}x^2e^{2c} - \frac{9}{2}\right)\cdot x^2$$

$$\boxed{y^2 = \frac{1}{2}x^4e^{2c} - \frac{9}{2}x^2}$$