Separable

Problem 5

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## Separable: Problem 5

(4 points) local/set3-Separable/problem5.pg

This set is **visible to students.**

Separate the following differential equation and integrate to find the general solution:
$y^2 e^{-6x} y' = 4x$

Then give the particular solution that satisfies the initial condition $y(0) = 5$ and state the interval on which this solution is valid.

General Solution (explicitly): $y(x) =$ ☐

Particular Solution (explicitly): $y(x) =$ ☐

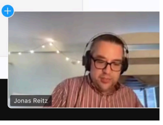
Interval of Validity: ☐

Hint:

Live Transcription (Closed Captioning) has been enabled

Who can see this transcript?

So, separate the following difference.




$y^2 e^{-6x} y' = 4x$ (divide both sides by $e^{-6x}$)

$y^2 y' = \frac{4x}{e^{-6x}}$ → move up, change sign of exponent

$\frac{4x}{e^{-6x}} \cdot \frac{e^{6x}}{e^{6x}} = \frac{4xe^{6x}}{1} = 4xe^{6x}$

$y^2 y' = 4xe^{6x}$

$\int y^2 y' \, dx = \int 4xe^{6x} dx$

$\frac{y^3}{3} = 4\int xe^{6x} dx$ (Int. by Parts)

$\int y^2 \frac{dy}{dx} dx$

$u = x \quad dv = e^{6x}dx$

$du = 1dx \quad v = \int e^{6x} dx = \frac{1}{6}e^{6x} + C$

u-sub: $u = 6x$, $du = 6dx$, $\frac{1}{6}du = dx$

sub: $\int e^u \cdot \frac{1}{6} du = \frac{1}{6}\int e^u du = \frac{1}{6}e^u + C = \frac{1}{6}e^{6x} + C$

aside: $\int \sin 6x = -\frac{1}{6}\cos 6x + C$

$\int xe^{6x} dx = u \cdot v - \int v \, du$

$= x \cdot \left(\frac{1}{6}e^{6x}\right) - \int \frac{1}{6}e^{6x} dx$

$= \frac{1}{6}xe^{6x} - \frac{1}{6}\left(\frac{1}{6}e^{6x} + C\right)$

$= \frac{1}{6}xe^{6x} - \frac{1}{36}e^{6x} - \frac{1}{6}C$

$= \boxed{\frac{1}{6}xe^{6x} - \frac{1}{36}e^{6x} + C_1}$

$\frac{y^3}{3} = 4\int xe^{6x} dx$

$\frac{y^3}{3} = 4\left(\frac{1}{6}xe^{6x} - \frac{1}{36}e^{6x} + C_1\right)$

$\frac{y^3}{3} = \frac{2}{3}xe^{6x} - \frac{1}{9}e^{6x} + 4C_1$

multiply by 3:

$y^3 = 2xe^{6x} - \frac{1}{3}e^{6x} + 12C_1$

$\sqrt[3]{y^3} = \sqrt[3]{2xe^{6x} - \frac{1}{3}e^{6x} + 12C_1}$

$$\boxed{y = \sqrt[3]{2xe^{6x} - \frac{1}{3}e^{6x} + C_2}}$$

Then give the particular solution that satisfies the initial condition $y(0) = 5$ and state the interval on which this solution is valid.

plug in $y(0) = 5$ $\quad (x = 0,\ y = 5)$

$$5 = \sqrt[3]{2\cdot 0 e^{6\cdot 0} - \frac{1}{3}e^{6\cdot 0} + C_2}$$

$$5 = \sqrt[3]{-\frac{1}{3} + C_2}$$

raise both sides to power 3:

$$5^3 = \left(\sqrt[3]{-\frac{1}{3} + C_2}\right)^3$$

$$5^3 = -\frac{1}{3} + C_2$$

$$+\frac{1}{3} \quad 125 = -\frac{1}{3} + C_2$$

$$125 + \frac{1}{3} = C_2$$

$$125\cdot\frac{3}{3} + \frac{1}{3} = C_2$$

$$\frac{375}{3} + \frac{1}{3} = C_2$$

$$\boxed{\frac{376}{3} = C_2}$$

particular solution:

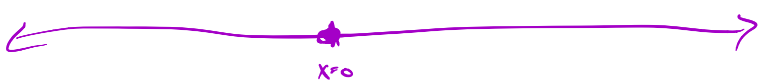
$$\boxed{y = \sqrt[3]{2xe^{6x} - \frac{1}{3}e^{6x} + \frac{376}{3}}}$$

Interval of validity?

1. what is the domain of the function?
2. Interval of validity is the largest interval in the domain that contains the initial condition.

Initial condition $y(0)=5$, has $x=0$.

Need interval containing $x=0$




Domain is $(-\infty, \infty)$

Interval of Validity is $(-\infty, \infty)$

Problem 1

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(4 points) local/set2-NonHomogeneousLinear/problem1.pg

This set is **visible to students.**

Find the general solution for the following differential equation:

$y' + 2y = 2e^{-t} - 5t$

$y(t) =$ ☐

complementary eq:

$y' + 2y = 0$

$\quad -2y \quad -2y$

$y' = -2y$

$\int \frac{y'}{y} dt = \int -2 \, dt$

$\int \frac{1}{y} \cdot y' dx = \int -2 \, dx$

$\ln|y| = -2t + C$

$e^{\ln|y|} = e^{-2t+C}$ choose $C=0$ b/c we want "a solution"

$|y| = e^{-2t}$

$y = \pm e^{-2t}$ choose +

$\boxed{y_1 = e^{-2t}}$ ← a solution to complementary eq.

STEP 2: guess $y = u \cdot y_1$

$\boxed{y = u \cdot e^{-2t}}$

$y' = u' e^{-2t} + u \cdot (-2) e^{-2t}$

$\boxed{y' = u' e^{-2t} - 2u e^{-2t}}$

$y' + 2y = 2e^{-t} - 5t$

substitute $y, y'$:

$u' e^{-2t} - \cancel{2ue^{-2t}} + \cancel{2ue^{-2t}} = 2e^{-t} - 5t$

$\frac{u' e^{-2t}}{e^{-2t}} = \frac{2e^{-t} - 5t}{e^{-2t}}$

$u' = \frac{2e^{-t}}{e^{-2t}} - \frac{5t}{e^{-2t}}$

$\int u' dt = \int 2e^{t} - 5te^{2t} \, dt$

$u = 2e^{t} - 5\int te^{2t} \, dt$

↑ use integration by parts

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Problem 3

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# Separable: Problem 3

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This set is **visible to students.**

Separate the following differential equation and integrate to find the general solution:

$y' = \cos^2(-5x)\cos^2(-3y)$

General Solution (implicitly): ☐

If you don't get this in 5 tries, you can get a hint.

Hint:

Edit

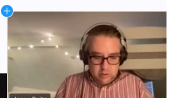
Show: ☐ CorrectAnswers ☐ ProblemGrader

Preview My Answers | Check Answers | Submit Answers

You have attempted this problem 0 times.

You have unlimited attempts remaining.

Okay, so let me give me just one second, I'm going to grab on my tablet a copy of it and go from there. So let's.



$y' = \cos^2(-5x)\cos^2(-3y)$

$\cos^2(-3y)$ $\qquad$ $\cos^2(-3y)$

$$\int \frac{1}{\cos^2(-3y)} y' dx = \int \cos^2(-5x)\, dx$$

$\frac{1}{\cos x} = \sec x$

$\frac{d}{dx} \tan x = \sec^2 x$

$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$

$$\int \sec^2(-3y) \cdot y' dx = \int \cos^2(-5x)\, dx$$

u-sub.

$$\frac{1}{-3}\tan(-3y) = \int \frac{1+\cos(2\cdot(-5x))}{2} dx$$

$$= \int \frac{1+\cos(-10x)}{2} dx$$

$$= \frac{1}{2}\int 1 + \cos(-10x)\, dx$$

$$-\frac{1}{3}\tan(-3y) = \frac{1}{2}\left(x + \frac{\sin(-10x)}{-10} + C\right)$$