WeBWork - Review

#7 Int. by parts

$\int u\,dv = uv - \int v\,du$

$$\int 3x\ln(2x)\,dx$$

~~$u = 3x$~~ ~~$dv = \ln(2x)\,dx$~~

~~$du = 3\,dx$~~ ~~$v = ??$ → looks complicated?~~

~~Aside: The derivative of $\ln x$ is $\frac{1}{x}$, not integral!~~

Try switching u, dv:

$u = \ln(2x)$ $dv = 3x$

$du = \frac{1}{2x}\cdot 2\,dx$ $v = \int 3x\,dx$

$du = \frac{1}{x}\,dx$ $v = \frac{3}{2}x^2$

$$\int 3x\ln(2x)\,dx = \overset{u\,\cdot\,v}{\ln(2x)\cdot\frac{3}{2}x^2} - \overset{v\,\cdot\,du}{\int \frac{3}{2}x^2\frac{1}{x}dx}$$

$$= \frac{3}{2}x^2\ln(2x) - \int \frac{3}{2}x\,dx$$

x'

$$= \frac{3}{2}x^2\ln(2x) - \frac{3}{2}\cdot\frac{1}{2}x^2 + C$$

$$\boxed{= \frac{3}{2}x^2\ln(2x) - \frac{3}{4}x^2 + C}$$

Today:

A. Solve homogeneous linear first order ~~linear~~ diffy q's

B. Solve nonhomogeneous linear first order ~~linear~~ diffy q's

y is a function of x

RECALL: Implicit differentiation - find the derivative $\frac{d}{dx}\ln|y|$

$\frac{d}{dx}x^3 = 3x^2$ $\qquad$ $\frac{d}{dx}y^3 = 3y^2\cdot\frac{dy}{dx}$

$\int 3x^2\,dx = \frac{3}{3}x^3 = x^3$ $\qquad$ $\int 3y^2\frac{dy}{dx}\,dx = 3\frac{1}{3}y^3 = y^3$

$$\frac{d}{dy}y^3 = 3y^2$$

$\frac{d}{dx}\ln x = \frac{1}{x}$, $\frac{d}{dx}\ln y = \frac{1}{y}\cdot\underbrace{\frac{dy}{dx}}_{=y'}$

$\int \frac{1}{x}\,dx = \ln|x| + C$ $\qquad$ $= \frac{1}{y}\cdot y'$

$$\int \frac{1}{y}\cdot y' = \ln|y| + C$$

Defn. the order of a differential equation is the highest derivative that appears

Defn: A first-order differential equation is **linear** if it has the form: $\boxed{y' + p(x)y = f(x)}$

We call such an equation **homogeneous** if $f(x) = 0$, that is: $y' + p(x)y = 0$

Example 1: Solve $y' - x^2y = 0$ First order. linear, homogeneous.

$p(x) = -x^2$

$f(x) = 0$

ANS: $y = C_1 e^{\frac{1}{3}x^3}$

First order linear homogeneous

1. separate variables
   - y's on left
   - x's on right
   - ys are connected to y' by multiplication
2. integrate to find y
3. solve for y if possible.

(if there is an initial condition, plug in to find constant C)

ex:

① $y' - x^2y = 0$

$\quad +x^2y \qquad +x^2y$

$\frac{y'}{y} = \frac{x^2y}{y}$

$\frac{1}{y}\cdot y' = x^2$

or

$\frac{y'}{y} = x^2$

② $\int \frac{y'}{y}\,dx = \int x^2\,dx$

$$\ln|y| = \frac{1}{3}x^3 + C$$

Find y

First order, linear $y' + p(x)y = f(x)$

Example 3: $y' + 2y = x^3 e^{-2x}$

$p(x) = 2$ $\quad f(x) = x^3 e^{-2x}$

STRATEGY to solve $y' + p(x)y = f(x)$:

STEP 1: Solve the complementary equation $y' + p(x)y = 0$, let $y_1(x)$ be a solution.

STEP 2: Look for a solution of the form $y = uy_1$. Substitute into the original equation.

STEP 3: Solve for u by isolating u' and integrating.

STEP 4: Substitute $y_1(x)$, $u(x)$ to find the final solution: $y = uy_1$

① complementary equation: $y' + 2y = 0$

y_1 is a solution to complementary eq.

② guess $y = u \cdot y_1$ u is an unknown fcn.

substitute y, y' (find y' first)

③ solve for u, integrate

④ $y = u \cdot y_1$

$$e^{\ln|y|} = e^{\frac{1}{3}x^3 + C}$$

$$|y| = e^{\frac{1}{3}x^3} \cdot \underbrace{e^C}_{C_1}$$

$$|y| = e^{\frac{1}{3}x^3} \cdot C_1$$

$$\boxed{y = \pm C_1 e^{\frac{1}{3}x^3}}$$ Ans

$$\boxed{y = C_2 e^{\frac{1}{3}x^3}}$$

☆ To be continued next class ☆

Office Hours

WW Review #8

$f(x) = x^3 + 2\sqrt{x}$ F(x) is antideriv.

$F(1) = -3$

find $F(x)$

$$F(x) = \int x^3 + 2x^{\frac{1}{2}}\, dx$$

$$F(x) = \frac{x^4}{4} + \frac{2x^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{x^4}{4} + \frac{2}{3} \cdot \frac{2x^{\frac{3}{2}}}{1} + C$$

$$F(x) = \frac{x^4}{4} + \boxed{\frac{4}{3} x^{\frac{3}{2}}} + C$$

$$\frac{x^4}{4} + \frac{4x^{3/2}}{3} + C$$

plug in $F(1) = -3$

$$-3 = \frac{1^4}{4} + \frac{4}{3} 1^{3/2} + C$$

$$-3 = \underbrace{\frac{1}{4} + \frac{4}{3}}_{} + C$$

$$-3 = \underbrace{\frac{3}{12} + \frac{16}{12}}_{} + C$$

$$-3 = \cancel{\frac{19}{12}} + C$$

$$-\frac{19}{12} \qquad \cancel{-\frac{19}{12}}$$

$$\underset{\frac{\cdot 12}{1 \cdot 12}}{-3} - \frac{19}{12} = C$$

$$\frac{-36}{12} - \frac{19}{12} = c$$

$$\frac{-55}{12} = c$$

$$F(x) = \boxed{\frac{x^4}{4} + \frac{4}{3}x^{\frac{3}{2}} - \frac{55}{12}}$$

#11 $f(t) = 6\sec^2 t - 7t^3$ $F(0)=0$, find $F(1.4)$

$$F(t) = 6\tan t - \frac{7t^4}{4} + C$$

plug in $F(0)=0$

$$0 = 6 \cdot \tan 0 - \frac{7 \cdot 0^4}{4} + c$$

$$0 = 6 \cdot 0 - 0 + c$$

$$0 = 0 + c$$

$$\boxed{0 = c}$$

$$\boxed{F(t) = 6\tan t - \frac{7t^4}{4}}$$

$$F(1.4) = 6\tan(1.4) - \frac{7(1.4)^4}{4}$$

ex9 $f(x) = x^2 + \sin x$, $\underline{\underline{F(\pi) = 0}}$

$$\downarrow$$

suppose we integrate and get:

$$F(x) = \frac{x^3}{3} - \cos x + C$$

plug in $F(\pi) = 0$

$$0 = \frac{\pi^3}{3} - \cos \pi + C$$

$$0 = \frac{\pi^3}{3} - (-1) + C$$

$$0 = \frac{\pi^3}{3} + 1 + C$$

$$-\frac{\pi^3}{3} - 1 \qquad -\frac{\pi^3}{3} - 1$$

$$\boxed{-\frac{\pi^3}{3} - 1 = C} \circledast$$

$$\boxed{C = -11.335426} \circledast$$

$$\boxed{F(x) = \frac{x^3}{3} - \cos x - \frac{\pi^3}{3} - 1}$$

$$\boxed{F(x) = \frac{x^3}{3} - \cos x - 11.33526}$$