Properties of relations. If R is a relation on set A,

1. R is <u>reflexive</u> if xRx for every $x \in A$, $(\forall x \in A,\ xRx)$
2. R is <u>symmetric</u> if xRy means yRx $(\forall x,y \in A,\ xRy \to yRx)$.
3. R is <u>transitive</u> if xRy and yRz then xRz $(\forall x,y,z \in A,\ (xRy \wedge yRz) \to xRz)$

Defn: an <u>equivalence relation</u> is a relation R that is <u>reflexive</u>, <u>symmetric</u> and <u>transitive</u>.

4. R is <u>antisymmetric</u> if xRy and yRx means $x=y$. $(\forall x,y \in A\ (xRy \wedge yRx) \to x$
5. R is <u>irreflexive</u> if $x \not R x$ for every $x \in A$ $(\forall x \in A,\ x \not R x)$.

Given the following relations on the set of all people. Check ALL correct answers fr
lists:

(a) a is older than b

- [ ] **A.** reflexive
- [x] **B.** antisymmetric
- [ ] **C.** symmetric
- [ ] **D.** transitive
- [x] **E.** irreflexive

(b) a and b have a common grandparent

- [ ] **A.** antisymmetric
- [ ] **B.** symmetric

<u>antisymmetric</u>

for members of our set, if (xRy and yRx) then $x=y$

<u>can we find two people a and b</u>

such that aRb "a is older than b"

<u>and</u> bRa "b is older than a"?

a = prof. Reitz

b = Taia

aRb? Yes

bRa? No

"if (aRb and bRa) then $a=b$"

is this true or false? TRUE

since antecedent is false.

This relation <u>IS</u> antisymmetric.

$<$

antisymmetric:

for all x and y, if ($x<y$ and $y<x$) then $x=y$

<u>IS antisymmetric</u>

$\neq$

antisymmetric:

for all x and y, if ($x \neq y$ and $y \neq x$) then $x = y$

False for any x, y that are not the same.

Not antisymmetric.

(b) a and b have a common grandparent

aRb = "a and b have a common grandparent"

antisymmetric

for all people a and b, if (aRb and bRa) then a = b.

a = Prof. Reitz

b = my sister

NOT antisymmetric

reflexive

a = Prof. Reitz

b = Prof. Reitz

aRb? Yes

bRa? Yes

$\boxed{aRa}$ Yes

IS REFLEXIVE.

Symmetric

for all a,b if aRb then bRa

Is Symmetric

Review topics:

Revision Test

#12

12. a. Let $A_1 = \{-1, 2\}$, $A_2 = \{-3, 4\}$, $A_3 = \{-5, 6\}$ and in general for each $n \in \mathbb{N}$, $A_n = \{-2n+1, 2n\}$. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$.

$A_1 = \{-1, 2\}$

$A_2 = \{-3, 4\}$

$A_3 = \{-5, 6\}$

$A_4 = \{-7, 8\}$

$A_n = \{-2n+1, 2n\}$

$A_5 = \{-9, 10\}$

what is $|A_1|$? $|A_1| = 2$

$\bigcup_{i=1}^{\infty} A_i =$ set of all negative odd numbers, and positive even numbers

$= \boxed{\{\ldots -5, -3, -1, 2, 4, 6, \ldots\}}$

$\bigcap_{i=1}^{\infty} A_i = \emptyset$

b. Let $\mathbb{R}^+$ be the set of positive real numbers. For each $\alpha \in \mathbb{R}^+$, let A_α be the closed interval $[0, \alpha]$. Find $\bigcup_{\alpha \in \mathbb{R}^+} A_\alpha$ and $\bigcap_{\alpha \in \mathbb{R}^+} A_\alpha$.

0.1 is not in $A_{0.01}$

$\alpha = 1.5$, $A_{1.5} = [0, 1.5]$

$\alpha = 2.5$, $A_{2.5} = [0, 2.5]$

$\alpha = 100$ $A_{100} = [0, 100]$

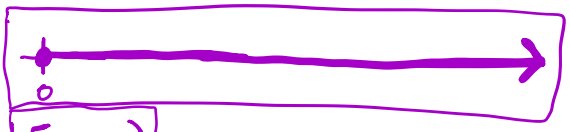
$\cancel{\alpha = 0}$ not allowed, $\alpha \in \mathbb{R}^+$

$|A_{1.5}| = \infty$

$|A_{2.5}| = \infty$

are these ∞'s equal?

$\bigcup_{\alpha \in \mathbb{R}^+} A_\alpha =$



$= \boxed{[0, \infty)}$

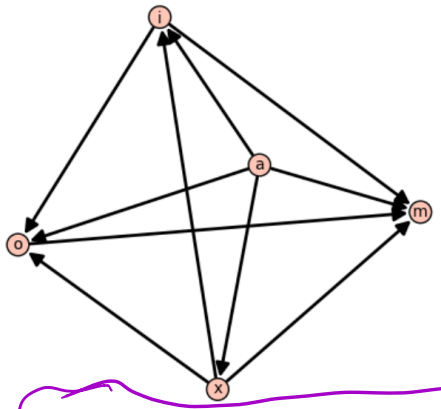
$\bigcap_{\alpha \in \mathbb{R}^+} A_\alpha = \boxed{\{0\}}$ $\cancel{\{\}}$

$= \boxed{[0, 0]}$ ok

16. a. The relation R on the set A is represented by the graph at right. Identify the sets A and R. Determine whether R is reflexive, symmetric, transitive, antisymmetric or irreflexive (explain why in each case).

b. Consider the relation S = { (3,3), (4,4), (4,5), (5,4), (5,5), (6,6), (6,7), (7,6), (7,7), (8,8)} on the set $B = \{3, 4, 5, 6, 7, 8\}$. Sketch a graph representing S. Is S an equivalence relation (justify your answer)? If so, list the equivalence classes.

~~c. Prove that the relation R on $\mathbb{Z}$ given by xRy if and only if $x \equiv y \bmod 5$ is an equivalence relation, and describe its equivalence classes.~~



vertices $A = \{a, i, m, o, x\}$

arrows $R = \{(a,i), (i,o), (x,o), (i,m), (x,m), (x,i), (x,o), (o,m), (a,o), (a,m)\}$

a loop on every point → reflexive? $\forall x \in A$, xRx

$(x,x) \in R$

Not reflexive - no element of A is related to itself.

every arrow goes both ways → Symmetric? $\forall x,y \in A$ if xRy then yRx

Not symmetric - for example aRi but $i \not R a$.

every 2-step path can be completed in 1-step → transitive? $\forall x,y,z \in A$ if (xRy and yRz) then xRz

Is transitive (can't find x,y,z with xRy and yRz but $x \not R z$.)

R has no (x,y) and (y,x), no double arrows, (only loops are ok) → antisymmetric.

$\forall x,y$ if (xRy and yRx) then $x=y$

$B = \{4,5,6\}$
$S = \{(4,5),(5,4)\}$

not transitive.

4 → 5

6

transitive

$x=4$
$y=5$
$z=4$

$4R5$ ✓
$5R4$ ✓
$4R4$? NO

If you are allowed

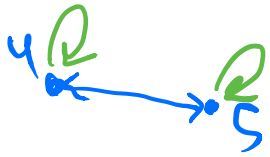
to add edges to S to make a new relation S^*, can you make S^* transitive?

no loops

irreflexive

$\forall x \quad x \not R x$

Yes, irreflexive.



6

This is transitive!

$$\boxed{S^* = \{(4,5)\ (5,4),\ (4,4)\ (5,5)\}}$$