Prop. for any $n \in \mathbb{N}$, the sum of the first n-many odd natural numbers equals n^2 — $P(n)$

all say the same thing

Prop $\forall n \in \mathbb{N}, \sum_{i=1}^{n} (2i-1) = n^2$

Prop $\forall n \in \mathbb{N}, 1+3+5+7+\ldots(2n-1) = n^2$

Proof (Induction)

Base case $n=1$. the sum of the first 1-many natural numbers is 1. $P(1)$
1^2 is also 1. Thus the statement is true for $n=1$.

Inductive case. Suppose $K \in \mathbb{N}$ — Suppose $P(K)$
and the sum of the first K-many odd natural numbers equals K^2.

So $1+3+5+7\ldots+(2K-1) = K^2$

$+2K+1$ $+2K+1$

$1+3+5+7+\ldots+(2k-1)+(2k+1)=k^2+2k+1$

$=(k+1)(k+1)$

$1+3+5+7+\ldots+(2k-1)+(2k+1)=(k+1)^2$

Thus, the sum of the first $k+1$ odd natural numbers equals $(k+1)^2$ $P(k+1)$

□

$k+1$ many odd natural #s

$$\overbrace{1+3+5+7+\ldots+(2k-1)+(2k+1)}=(k+1)^2$$

$k=6$ $\quad 1+3+5+7+9+11+13$

kth odd natural

Suppose $P(k)$

$\vdots$

Thus $P(k+1)$

$k=6$

$1+3+5+7+9+11 = 6^2$

$36 = 36$

what is $P(n)$?

Example Prop. for every natural number n,

$$1\cdot 2 + 2\cdot 3 + 3\cdot 4 + 4\cdot 5 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

Outline

Proof Base case: prove $P(1)$ holds

Inductive case: suppose $k \in \mathbb{N}$ and $P(k)$

$\vdots$

Thus $P(k+1)$

check $n=7$

$1\cdot 2 + 2\cdot 3 + 3\cdot 4 + 4\cdot 5 + 5\cdot 6 + 6\cdot 7 + 7\cdot 8$

$= \frac{7(8)(9)}{3}$ ✓

so

$2 + 6 + 12 + 20 + 30 + 42 + 56$

$168 = 168$ ✓

Example Prop. for every natural number n,

$$1\cdot 2 + 2\cdot 3 + 3\cdot 4 + 4\cdot 5 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3} \quad P(n)$$

Proof (Induction)

Base case. $n=1$, the left side equals $1\cdot 2 = 2$ $P(1)$

the right side equals $\frac{1(1+1)(1+2)}{3} = \frac{1\cdot 2\cdot 3}{3} = 2$

Thus the statement is true when $n=1$ □

Inductive case. Suppose $k \in \mathbb{N}$ and $P(k)$

$$1\cdot 2 + 2\cdot 3 + 3\cdot 4 + 4\cdot 5 + \dots + k(k+1) = \frac{k(k+1)(k+2)}{3}$$

$+(k+1)(k+2)$ $+(k+1)(k+2)$

$$1\cdot 2 + 2\cdot 3 + \dots + k(k+1) + (k+1)(k+2) = \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$$

$$= \frac{(k+1)(k+2)(k+3)}{3}$$

n $(k+1)$ $(k+2)$ $P(k+1)$

Thus

$$1\cdot 2 + 2\cdot 3 + 3\cdot 4 + 4\cdot 5 + \dots + (k+1)((k+1)+1) = \frac{(k+1)((k+1)+1)((k+1)+2)}{3}$$

$5\cdot 6 + 6\cdot 7 + \dots$ $k(k+1)$

□

$1\cdot 2 + 2\cdot 3 + 3\cdot 4 + 4\cdot 5 + 5\cdot 6 + \dots + k(k+1) + (k+1)((k+1)+1) =$

$(k+1)(k+2)$

equals the above — add this on.

$$= \frac{(k+1)(k+2)(k+3)}{3}$$

$$\frac{k(k+1)(k+2)}{3} + \frac{3(k+1)(k+2)}{3}$$

$$= \frac{k(k+1)(k+2) + 3(k+1)(k+2)}{3}$$

factor out $(k+1)(k+2)$

$$= \frac{(k+1)(k+2)(k+3)}{3}$$

Strong Induction

Outline of proof by strong induction

Prop. $\forall n \in \mathbb{N},\ P(n)$

Proof. [Base step. Prove $P(1)$. Sometimes we have two or more base steps, ex: $P(1)$ and $P(2)$

Inductive step for $k \in \mathbb{N}$, prove

$$P(1) \wedge P(2) \wedge P(3) \wedge \dots \wedge P(k) \rightarrow P(k+1)$$

Here, we assume $P(n)$ holds for all numbers upto and including k,

(regular induction: we assume $P(n)$ holds for $n = k$ only)